



# United Kingdom Input-Output Analytical Tables 2005

Editor: **Markus G Šova**

**Office for National Statistics**

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## Contacts

### This publication

For information about the content of this publication, contact Markus G Šova

Tel: 01633 455622

Email: [markus.sova@ons.gsi.gov.uk](mailto:markus.sova@ons.gsi.gov.uk)

### Other customer enquiries

ONS Customer Contact Centre

Tel: 0845 601 3034

International: +44 (0)845 601 3034

Minicom: 01633 815044

Email: [info@statistics.gsi.gov.uk](mailto:info@statistics.gsi.gov.uk)

Fax: 01633 652747

Post: Room 1.101, Government Buildings,  
Cardiff Road, Newport, South Wales NP10 8XG

[www.ons.gov.uk](http://www.ons.gov.uk)

### Media enquiries

Tel: 0845 604 1858

Email: [press.office@ons.gsi.gov.uk](mailto:press.office@ons.gsi.gov.uk)

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## Preface

The 2005 United Kingdom Input-Output Analytical Tables (IOATs) are a National Accounts product of the Office for National Statistics. These tables are consistent with the 2009 editions of United Kingdom National Accounts: The Blue Book (ONS, 2009a) and United Kingdom Balance of Payments: The Pink Book (ONS, 2009b).

The 2005 IOATs are derived from the 2005 annual Supply and Use Tables (SUTs). The SUTs provide a picture of the flows of products and services in the economy for a single year and are used to set the level of annual current price Gross Domestic Product (GDP). They show the composition of uses and resources across institutional sectors and the inter-dependence of industries in order to reconcile the production, income and expenditure approaches to the measurement of GDP.

The IOATs show separately the consumption of domestically produced and imported goods and services, providing a theoretical framework for further analysis of the structure of the economy, its composition and the effect of changes in final demand on the economy. These tables form an essential tool for economic modelling.

The first IOATs for the United Kingdom covered the year 1954. Since then, they have been produced roughly every 5 years, see Mahajan (2006a, pp306-308).

The presentation of the SUTs and the IOATs is based on the European System of Accounts 1995 (ESA 95: Eurostat, 1996), which itself is based on the United Nations System of National Accounts 1993 (SNA 93: United Nations, 1993). SNA 93 has been adopted worldwide and the production of ESA 95 based accounts is a legal requirement of European Union member states.

The last set of UK IOATs covered the year 1995. There have been several methodological improvements to the SUTs since the 1995 IOATs were compiled (see Mahajan, 2006b, and Akers and Clifton-Fearnside, 2008). An important example is the allocation of FISIM (Financial Intermediation Services Indirectly Measured) by industry, by sector and by final demand. Because of these differences, comparisons between the two sets of tables may be misleading.

All the monetary value estimates in the tables are calculated as accurately as possible, however they cannot be regarded as being accurate to the number of digits shown. Several revisions of the underlying SUTs are produced after the first set of tables has been compiled.

## Introduction

This publication describes the 2005 Input-Output Analytical Tables (IOATs) derived from the Supply and Use Tables (SUTs) for the same year, as published in ONS (2009a). These tables, together with supplementary data and certain economic assumptions, have been combined to construct the input-output table. This table describes how products (and primary inputs) are used to produce further products and satisfy final demand. The input-output table and its derivative outputs collectively form the IOATs. Their derivation and construction are described in the following sections of this publication.

The 2005 IOATs have been derived from SUTs compiled using 108 industry input-output groups (IOGs) consistent with the UK's Standard Industrial Classification 2003 (SIC (03)) for industries and Eurostat's Classification of Products by Activity (CPA (02)) for products, extended to 123 IOGs by separating components of the non-market output produced by general government and NPISHs (Non-Profit Institutions Serving Households) from the output produced by the market sectors, in order to allow for their different roles in the economy.

The close relationship between SUTs and IOATs has sometimes led to confusion despite being different products. This problem has been exacerbated by inconsistent use of terminology. This publication adheres to the terminology of Eurostat (2008).

This report contains a description of the theory and assumptions behind the 2005 IOATs and their practical implementation, together with summary IOATs. It is accompanied by spreadsheets containing both summary and detailed IOATs. It should be noted that some of the tables needed for the compilation of the IOATs cannot be published in order to adhere to the Code of Practice for Official Statistics (see UK Statistics Authority, 2009, principle 5: Confidentiality).

This report focuses on product by product tables. Some users may wish to use industry based analyses instead of the product based analyses shown here. We therefore conclude this publication with a description of an alternative way of performing industrial analyses based on the product by product tables.

The tables presented in this product, including summary and detailed tables in Microsoft Excel format, are provided free of charge at

[www.statistics.gov.uk/about/methodology\\_by\\_theme/inputoutput/latestdata.asp](http://www.statistics.gov.uk/about/methodology_by_theme/inputoutput/latestdata.asp) .

## Symbols and conventions

In general, the following symbols are used throughout this publication:

“-” is used to denote nil

“0” represents less than £0.5 million but more than nil

£1 billion denotes £1,000 million

In the summary and detailed tables, the sum of components may not equal the total due to rounding differences. As a result of these rounding differences, totals in different tables may differ slightly.

## Acknowledgements

The author would like to acknowledge and thank a number of people for their important contributions to this project: Chrisoulla Kirri and Yolanda Ruiz who did much of the data compilation and quality checks; Roger Akers for all his advice; the Input-Output team at the Scottish Government, especially Janet Lauchlan and Stevan Croasdale, for their time and assistance; colleagues in ONS and HMRC (Her Majesty’s Revenue and Customs) who have helpfully provided data and information beyond what is normally required of them; and Sanjiv Mahajan for his expertise and generosity of time. To all of the above the author is very grateful.

## Transforming Supply and Use Tables to Basic Prices

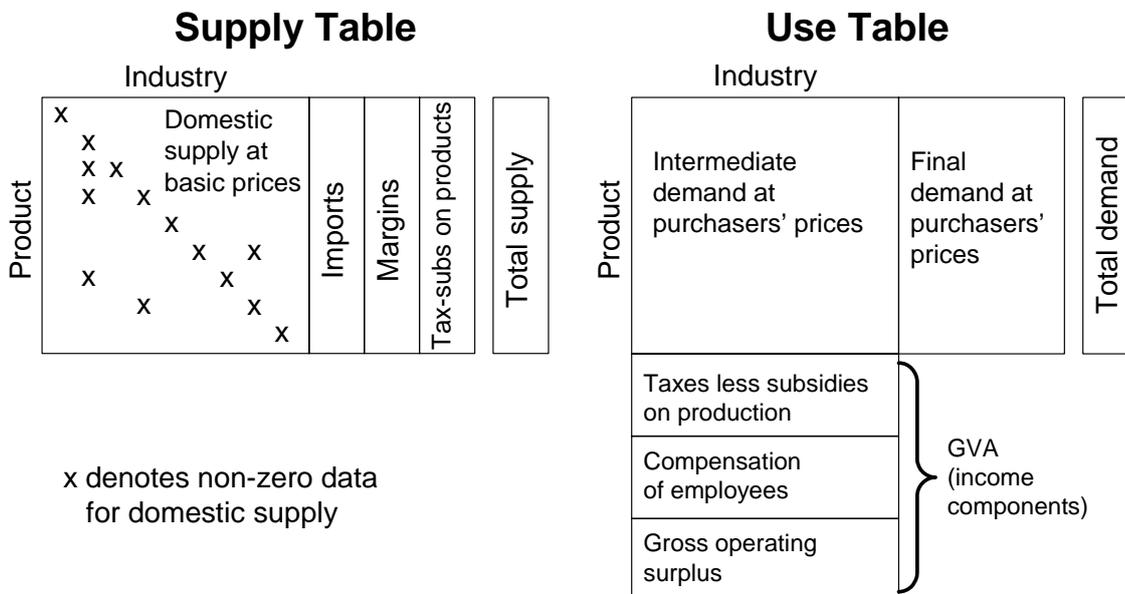
The annual SUTs consist of two matrices, which bring together the *production*, *income* and *expenditure* approaches to measuring of GDP (see ONS, 2009a, for the 2005 SUTs). Being balanced, they provide a single estimate of GDP, which integrates the components of value added, output and final demand. Their structures are shown in figure 1, and we note the following:

- The SUTs show the supply and demand for all products. The products are classified into 123 product input-output groups (IOGs) consistent with Eurostat’s Classification of Products by Activity (CPA (02)). Producers are similarly classified into 108 industry IOGs consistent with the UK Standard Industrial Classification 2003 (SIC (03)), which is itself consistent with Eurostat’s NACE Rev. 1.1 (*Nomenclature statistique des Activités économiques dans la Communauté Européenne*). The IOG definitions are listed in Annex A.
- The supply table shows the output of each product IOG by each industry IOG. As producers are classified according to their principal product, most domestic production lies on the diagonal. However, there are some off-diagonal elements in this table. These represent secondary production and by-products classified to IOGs other than the principal product of the industry. The supply table is relatively sparse because most producers produce a limited range of

products. The main body of the supply table (which shows domestic supply at basic prices – the value received by the producers) is not published due to the data being disclosive. The columns on the right of the supply table show imports of products, distributors’ trade margins on products and taxes less subsidies on products. Summing across these columns and those in the main body gives the total supply of products at purchasers’ prices, the value paid by purchasers excluding any refundable VAT.

- The main body of the use table shows, for each industry IOG, the intermediate demand for products. That is, the value of products used-up or altered by the production process. The columns to the left of the main table give the components of final demand for products. Both final demand and intermediate demand are valued at purchasers’ prices and cover domestically produced and imported products.
- The rows underneath the main body of the use table give the income components of Gross Value Added (GVA) for each industry IOG. These components are labour costs, taxes less subsidies on production, profits, etc.
- The construction of the product by product Input-Output table involves matrix operations which require the main bodies of the supply and use tables at basic prices to be square matrices. Therefore the SUTs at purchasers’ prices have had their product detail reduced to 108 product IOGs. These tables have then been extended to 123 product and industry IOGs by separating components of the non-market output produced by general government and NPISHs (Non-Profit Institutions Serving Households) from the output produced by the market sectors, in order to allow for their different roles in the economy. The CPA (02) and SIC (03) definitions of IOGs are given in Annex A.

**Figure 1: Supply and Use tables at purchasers’ prices**



Once SUTs have been compiled, the first stage in deriving IOATs is to transform the SUTs from purchasers’ prices to basic prices. This is achieved by adjusting for imports, margins and taxes less subsidies on products. For the supply table this is a simple task, because the components to be removed are contained in the columns to the right of the main body of the table. For the use

table the task is more complicated; it requires the construction of a use table for each of the components to be adjusted. The sum of these use matrices is called the *transition matrix*, it is subtracted from the use table at purchasers' prices to give the use table at basic prices. With the exception of distributors' trade and transport margins, these components are shown as separate rows in the primary inputs, leaving the industry totals unchanged. This is not needed for margins as they are reallocated across the goods consumed. The transition from purchasers' prices to basic prices is depicted by figure 2; the outcome is shown in figure 3. Tables 1 and 2 show summary 2005 domestic and imports use tables, respectively.

**Figure 2: SUT transition from purchasers' prices to basic prices**

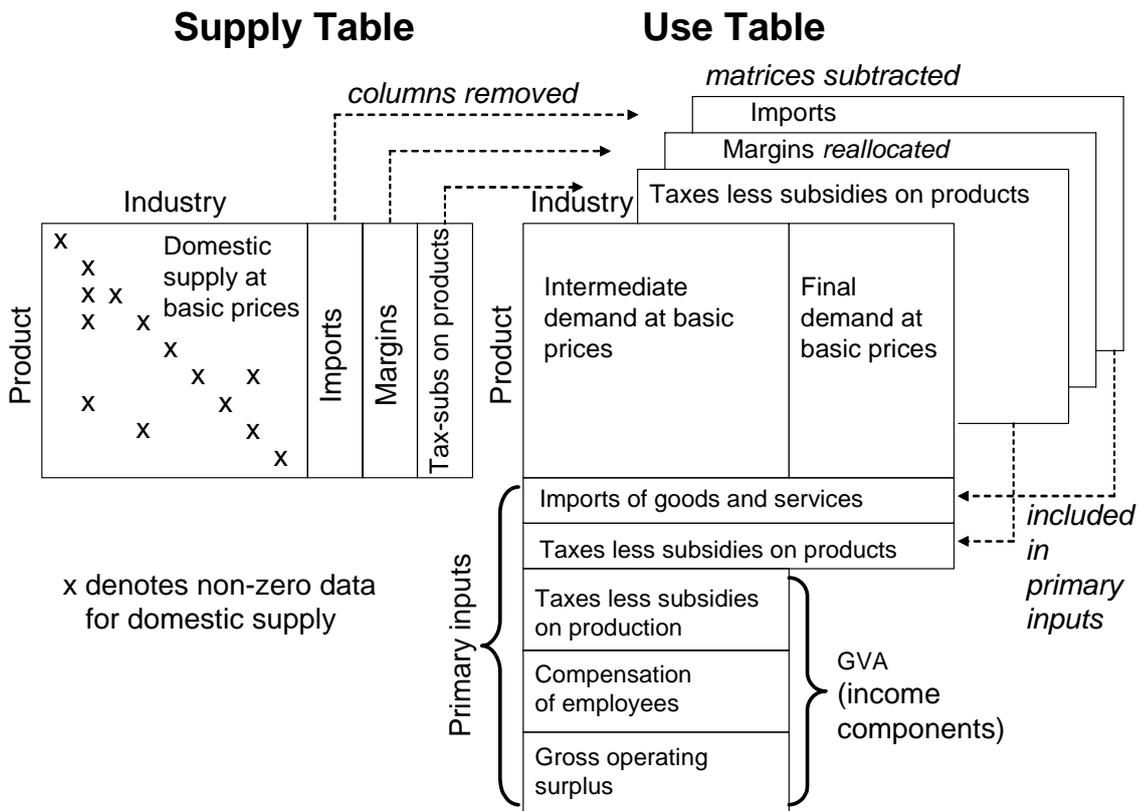


Table 1: Summary Domestic Use table at basic prices, 2005

£ million											
Product	Intermediate consumption by industries										
	1	2	3	4	5	6	7	8	9	10	11
	Agriculture [1-3]	Mining and quarrying [4-7]	Manufacturing [8-84]	Electricity, Gas and water supply [85-87]	Construction [88]	Wholesale and retail trade [89-92]	Transport and communication [93-99]	Financial intermediation [100-114]	Public administration [115]	Education, health and social work [116-118]	Other services [119-123]
1 Agriculture	1555	1	8813	0	206	1287	17	0	-	120	4
2 Mining and quarrying	3	2406	4080	15329	2395	46	72	2	-	3	39
3 Manufacturing	4690	1350	85071	2695	22397	20635	11925	8586	2542	3074	3255
4 Electricity, gas and water supply	446	764	9805	15599	282	2199	953	1550	196	318	323
5 Construction	255	700	1344	965	52625	1721	2149	11590	773	125	204
6 Wholesale & retail trade	1466	604	26198	649	4627	10450	3911	6562	201	768	945
7 Transport and communication	427	1227	12981	458	1298	31685	31747	24015	789	1478	2016
8 Financial intermediation	2106	2577	29266	3673	22032	54715	27267	128152	3130	4880	10124
9 Public administration	12	22	557	62	383	246	1962	6807	43	71	65
10 Education, health and social work	167	30	1146	211	170	974	1151	4715	743	3148	832
11 Other services	220	95	2928	183	160	1806	1474	3595	654	1429	8419
12 Public administration (non-market)	-	-	-	-	-	-	-	-	-	-	-
13 Education, health and social work (non-market)	-	-	-	-	-	-	-	-	-	-	-
14 Other services (non-market)	-	-	-	-	-	-	-	-	-	-	-
15 Financial intermediation (NPISH)	-	-	-	-	-	-	-	-	-	-	-
16 Education, health and social work (NPISH)	-	-	-	-	-	-	-	-	-	-	-
17 Other services (NPISH)	-	-	-	-	-	-	-	-	-	-	-
Total consumption	11347	9775	182191	39823	106574	125764	82627	195575	9071	15412	26227
Imports of goods and services	821	2107	93058	2352	3100	21719	13645	20259	1041	979	5689
Taxes less subsidies on products	440	190	2898	949	421	5636	3245	8574	-	1036	899
Taxes less subsidies on production	-3215	232	2334	1117	610	8300	1237	1897	-	282	1055
Compensation of employees	3515	3206	105247	4857	33320	104180	54517	154313	6091	19618	27558
Gross operating surplus	7230	24015	40530	10711	35938	50232	25135	182372	965	15284	19929
Total output	20138	39526	426259	59808	179962	315831	180406	562989	17168	52610	81357

Product	Non-market - Industry			NPISHs - Industry			Total intermediate demand
	12	13	14	15	16	17	
	Public administration (non-market) [115]	Education, health and social work (non-market) [116-118]	Other services (non-market) [119-123]	Financial intermediation (NPISH) [100-114]	Education, health and social work (NPISH) [116-118]	Other services (NPISH) [119-123]	
1 Agriculture	-	38	-	1	64	8	12115
2 Mining and quarrying	-	3	-	0	1	6	24386
3 Manufacturing	8726	9582	968	27	1418	428	187369
4 Electricity, gas and water supply	838	1178	234	6	143	30	34862
5 Construction	3665	738	377	7	52	30	77319
6 Wholesale & retail trade	3046	5261	293	14	398	125	65518
7 Transport and communication	3618	2870	447	38	652	244	115990
8 Financial intermediation	12636	9640	1943	303	2049	1338	315832
9 Public administration	181	-	2	1	19	10	10443
10 Education, health and social work	3502	25449	58	24	1309	87	43715
11 Other services	2877	1203	3836	11	570	1462	30921
12 Public administration (non-market)	-	-	-	-	-	-	-
13 Education, health and social work (non-market)	-	-	-	-	-	-	-
14 Other services (non-market)	-	-	-	-	-	-	-
15 Financial intermediation (NPISH)	-	-	-	-	-	-	-
16 Education, health and social work (NPISH)	-	-	-	-	-	-	-
17 Other services (NPISH)	-	-	-	-	-	-	-
Total consumption	39089	55961	8159	431	6675	3768	918469
Imports of goods and services	6410	9177	1192	93	359	471	182472
Taxes less subsidies on products	7899	6519	1042	5	331	89	40172
Taxes less subsidies on production	-	-	-	-	-	-	13849
Compensation of employees	45762	85379	4370	610	20734	4201	677478
Gross operating surplus	7248	3239	475	477	1168	373	425321
Total output	106409	160275	15238	1616	29267	8902	2257761

Product	Final consumption expenditure		Gross capital formation			Exports		Total demand at purchasers' prices	
	Households	NPISHs	General government	GFCF	Valuables	Changes in inventories	Exports of goods		Exports of services
	1 Agriculture	4325	-	-	481	-	-84		1509
2 Mining and quarrying	136	-	-	178	-	103	12169	229	37201
3 Manufacturing	47562	-	-	17122	-997	1778	135511	3123	391468
4 Electricity, gas and water supply	21810	-	-	-	-	-5	98	70	56835
5 Construction	6202	-	-	97154	-	1081	-	640	182396
6 Wholesale & retail trade	231103	-	-	6021	385	-7	21141	11365	335526
7 Transport and communication	38958	-	-	891	15	-14	-	18712	174551
8 Financial intermediation	175669	-	-	32117	2	319	613	74046	598599
9 Public administration	3626	-	-	2036	-	-	-	1063	17168
10 Education, health and social work	23865	-	-	-	-	83	-	1990	69623
11 Other services	36802	-	-	2376	-	-81	3577	3459	77054
12 Public administration (non-market)	-	-	106409	-	-	-	-	-	106409
13 Education, health and social work (non-market)	-	-	149184	-	-	-	-	-	149184
14 Other services (non-market)	-	-	12495	-	-	-	-	-	12495
15 Financial intermediation (NPISH)	-	1160	-	-	-	-	-	-	1160
16 Education, health and social work (NPISH)	-	22864	-	-	-	-	-	-	22864
17 Other services (NPISH)	-	6800	-	-	-	-	-	-	6800
Total consumption	590058	30824	268088	158376	-595	3174	174618	114750	2257761
Imports of goods and services	116284	-	-	39594	218	1442	31207	2424	373641
Taxes less subsidies on products	77799	-	-	11788	-	-144	5784	2012	137410
Total	784140	30824	268088	209758	-377	4472	211608	119186	2768812

**Table 2: Summary Imports Use table at basic prices  
(product by industry), 2005**

Product	Intermediate consumption by industries											£ million
	1	2	3	4	5	6	7	8	9	10	11	
	Agriculture	Mining and quarrying	Manufacturing	Electricity, Gas and water supply	Construction	Wholesale and retail trade	Transport and communication	Financial intermediation	Public administration	Education, health and social work	Other services	
	[1-3]	[4-7]	[8-84]	[85-87]	[88]	[89-92]	[93-99]	[100-114]	[115]	[116-118]	[119-123]	
1 Agriculture	378	0	541	7	7	331	22	11	-	2	24	
2 Mining and quarrying	1	904	13286	1566	25	155	12	10	-	0	4	
3 Manufacturing	267	639	72867	381	2065	16677	5518	2745	939	660	1266	
4 Electricity, gas and water supply	20	3	70	151	3	24	11	18	1	3	3	
5 Construction	0	38	24	0	383	1	1	63	0	0	0	
6 Wholesale & retail trade	21	28	472	46	133	1203	564	803	78	86	90	
7 Transport and communication	18	115	1199	5	25	386	6300	626	11	27	69	
8 Financial intermediation	87	373	4519	180	454	2828	879	15523	9	104	3437	
9 Public administration	-	-	-	-	-	-	-	-	-	-	-	
10 Education, health and social work	8	1	29	5	3	16	26	188	2	57	14	
11 Other services	19	6	53	10	0	97	312	271	1	39	781	
12 Public administration (non-market)	-	-	-	-	-	-	-	-	-	-	-	
13 Education, health and social work (non-market)	-	-	-	-	-	-	-	-	-	-	-	
14 Other services (non-market)	-	-	-	-	-	-	-	-	-	-	-	
15 Financial intermediation (NPISH)	-	-	-	-	-	-	-	-	-	-	-	
16 Education, health and social work (NPISH)	-	-	-	-	-	-	-	-	-	-	-	
17 Other services (NPISH)	-	-	-	-	-	-	-	-	-	-	-	
Total consumption	821	2107	93058	2352	3100	21719	13645	20259	1041	979	5689	

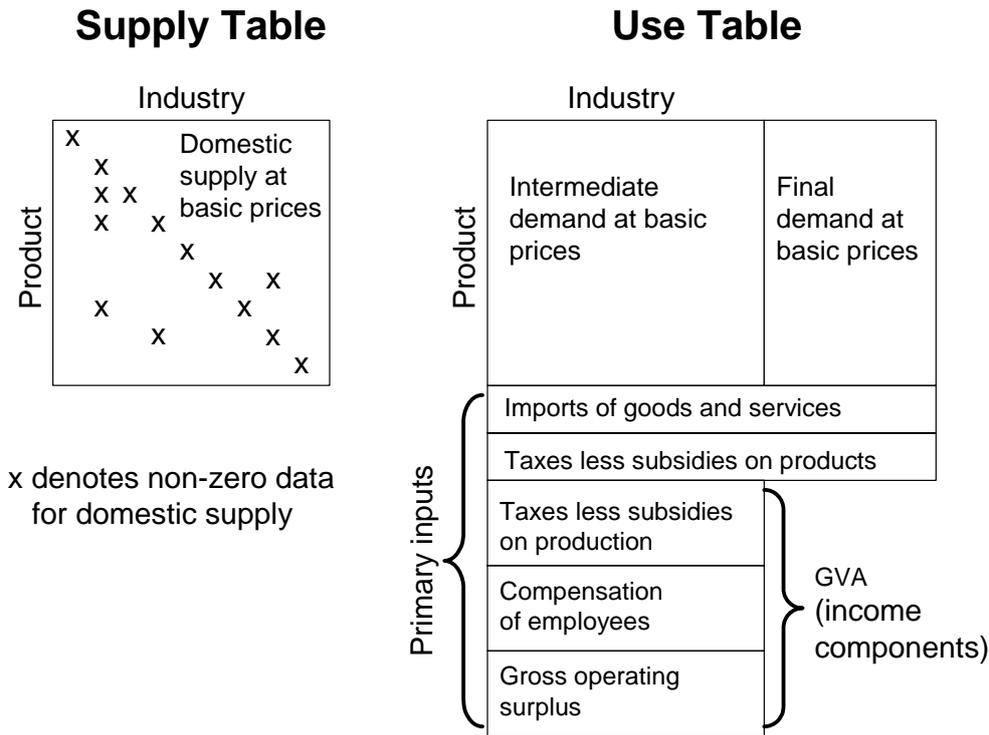
  

Product	Non-market - Industry			NPISHs - Industry			Total intermediate demand
	12	13	14	15	16	17	
	Public administration (non-market)	Education, health and social work (non-market)	Other services (non-market)	Financial intermediation (NPISH)	Education, health and social work (NPISH)	Other services (NPISH)	
	[115]	[116-118]	[119-123]	[100-114]	[116-118]	[119-123]	
1 Agriculture	-	0	-	-	0	0	1323
2 Mining and quarrying	-	-	-	-	0	0	15964
3 Manufacturing	4654	6794	43	6	219	62	115804
4 Electricity, gas and water supply	15	26	4	0	2	0	354
5 Construction	-	-	-	0	-	0	512
6 Wholesale & retail trade	12	29	2	0	1	1	3570
7 Transport and communication	324	144	42	1	17	10	9318
8 Financial intermediation	1127	1961	706	83	117	284	32670
9 Public administration	44	-	-	-	-	0	44
10 Education, health and social work	124	177	1	1	1	0	654
11 Other services	112	47	395	1	3	113	2260
12 Public administration (non-market)	-	-	-	-	-	-	-
13 Education, health and social work (non-market)	-	-	-	-	-	-	-
14 Other services (non-market)	-	-	-	-	-	-	-
15 Financial intermediation (NPISH)	-	-	-	-	-	-	-
16 Education, health and social work (NPISH)	-	-	-	-	-	-	-
17 Other services (NPISH)	-	-	-	-	-	-	-
Total consumption	6410	9177	1192	93	359	471	182472

Product	Final consumption expenditure		Gross capital formation			Exports		Total demand at purchasers' prices	
	Households	NPISHs	General government	GFCF	Valuables	Changes in inventories	Exports of goods		
							Exports of services		
1 Agriculture	5723	-	-	136	-	-25	-	7157	
2 Mining and quarrying	28	-	-	213	-	187	-	20367	
3 Manufacturing	82937	-	-	38378	235	1279	-	265863	
4 Electricity, gas and water supply	121	-	-	-	-	-	-	475	
5 Construction	124	-	-	-	-	-	-	636	
6 Wholesale & retail trade	10416	-	-	-	-	-	-	13986	
7 Transport and communication	8578	-	-	180	-15	-	2424	20485	
8 Financial intermediation	1927	-	-	577	-2	2	-	35173	
9 Public administration	-	-	-	-	-	-	-	44	
10 Education, health and social work	1347	-	-	-	-	-	-	2001	
11 Other services	5084	-	-	110	-	-	-	7454	
12 Public administration (non-market)	-	-	-	-	-	-	-	-	
13 Education, health and social work (non-market)	-	-	-	-	-	-	-	-	
14 Other services (non-market)	-	-	-	-	-	-	-	-	
15 Financial intermediation (NPISH)	-	-	-	-	-	-	-	-	
16 Education, health and social work (NPISH)	-	-	-	-	-	-	-	-	
17 Other services (NPISH)	-	-	-	-	-	-	-	-	
Total	116284	-	-	39594	218	1442	31207	2424	373641

**Figure 3: The outcome of the SUT transition to basic prices**



## The Input-Output Table

The Input-Output table (IOT) describes how products (and primary inputs) are used to produce further products and to satisfy final demand. To explain how this is achieved, let us first consider the following definitions of the domestic supply and use tables at basic prices:

- The (product by industry) domestic supply table at basic prices shows the outputs of the production process of each industry IOG.
- The (product by industry) domestic use table at basic prices shows the inputs of the production process of each industry IOG and each component of final demand.

The IOT is constructed by transforming the industry IOGs to product IOGs, so that these definitions become:

- The transformed (product by product) domestic supply table at basic prices shows the outputs of the production process of each product IOG.
- The transformed (product by product) domestic use table at basic prices shows the inputs of the production process of each product IOG and each component of final demand.

These transformed tables are depicted in figure 4.

As the output of the production process of a product is simply that product, the transformed domestic supply table is a diagonal matrix. The non-zero elements of this matrix are simply the total domestic supply of each product (at basic prices).

The transformed domestic use table at basic prices is the Input-Output Table. Due to the fact that the production of a product may require many inputs (such as raw materials, energy, office costs, accountancy, legal fees, etc) the IOT is not diagonal.

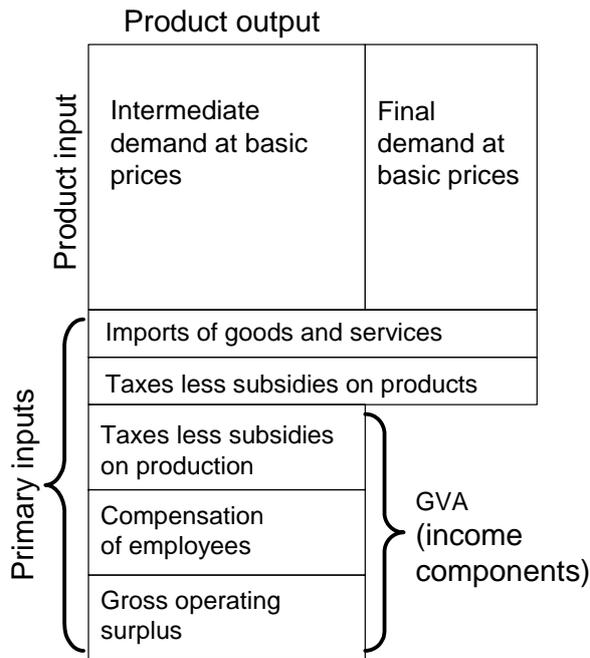
**Figure 4: The result of the transformation to product by product tables**

**Transformed Supply**



x denotes non-zero data for domestic supply

**Input-Output Table**



The product by industry domestic supply table describes the relationship between the production processes of industry IOGs and the production process producing product IOGs. The transformation of the supply table involves moving off-diagonal entries onto the industry IOG of which they are the principal product. The total supply of each product IOG remains unaltered; each element of off-diagonal supply is moved to the diagonal element of the same row. For each element of supply thus moved, its corresponding inputs (including primary inputs) need to be moved in the use table. This is complicated by the fact the production of a product may require many inputs. In essence, for each industry IOG we need to separate out the inputs for the various product IOGs produced by that industry. It would be impractical for producers to provide such splits, therefore this is estimated by making a *technology assumption*. Three technology assumptions are described in Eurostat (2008, ch11):

- **Product technology** assumes that all products classified to a product IOG have the same input structure, regardless of the classification of the producer. Unfortunately, it can result in negative entries in the main body of the IOT, even though such negative intermediate use entries are conceptually impossible.
- **Industry technology** assumes that all products produced by an industry IOG have the same input structure, regardless of the classification of the product. This is conceptually less satisfactory than the product technology assumption. However, it does not result in negative entries in the main body of the IOT.

- It is possible to specify either product technology or industry technology independently for each off-diagonal entry in the supply table. This **hybrid technology** matrix can result in negative entries, but these can be avoided through an appropriate choice of technology for each off-diagonal entry.

Each of these technology assumptions generates a set of simultaneous equations which can be solved by matrix algebra. This is not easy to visualise numerically; the interested reader is referred to Eurostat (2008, ch11). The negative entries can have a number of root causes:

- The product technology assumption can be inappropriate for products which can be produced by more than one method (such as electricity produced by hydro and nuclear power stations).
- The set of products classified to a product IOG may be heterogeneous in terms of their inputs (such as beer and cider).
- Different producers of the same product may vary in what production they do in-house. Consider, for example, two bakeries. The first buys flour to make bread. The second buys grain to make flour to make identical bread. Thus the inputs to the first bakery's loaves include flour but no grain. The inputs to the second bakery's loaves include grain but little flour.
- There may be quality issues in the data, either in the SUTs or in the transition matrix.

For the 2005 IOT, a hybrid technology assumption has been used. The matrix algebra is given in Annex B. Table 3 shows the summary 2005 Input-Output table.

**Table 3: Summary Input-Output table (product by product), 2005**

£ million											
Product	Intermediate consumption by products										
	1	2	3	4	5	6	7	8	9	10	11
	Agriculture [1-3]	Mining and quarrying [4-7]	Manufac- turing [8-84]	Electricity, Gas and water supply [85-87]	Construc- tion [88]	Wholesale and retail trade [89-92]	Transport and com- munication [93-99]	Financial intermed- iation [100-114]	Public adminis- tration [115]	Education, health and social work [116-118]	Other services [119-123]
1 Agriculture	1366	2	8518	0	218	1616	28	86	-	137	53
2 Mining and quarrying	3	2223	3867	14638	2516	281	231	579	-	4	38
3 Manufacturing	4320	1265	78116	2550	22618	24468	11687	12581	2542	3961	3305
4 Electricity, gas and water supply	411	710	9170	14751	465	2691	1111	2313	196	421	349
5 Construction	228	663	1250	921	52495	1890	2032	11900	773	195	255
6 Wholesale & retail trade	1347	563	24167	615	4680	11613	3855	7706	201	1064	953
7 Transport and communication	385	1125	11981	437	1548	32421	30585	25589	789	1838	1940
8 Financial intermediation	1943	2435	26526	3508	22401	56132	26296	131799	3130	5994	9644
9 Public administration	10	20	509	59	391	221	1934	6930	43	76	48
10 Education, health and social work	154	29	1051	201	200	1087	1094	4877	743	6544	779
11 Other services	201	89	2467	176	198	2141	1410	4971	654	1678	8357
12 Public administration (non-market)	-	-	-	-	-	-	-	-	-	-	-
13 Education, health and social work (non-market)	-	-	-	-	-	-	-	-	-	-	-
14 Other services (non-market)	-	-	-	-	-	-	-	-	-	-	-
15 Financial intermediation (NPISH)	-	-	-	-	-	-	-	-	-	-	-
16 Education, health and social work (NPISH)	-	-	-	-	-	-	-	-	-	-	-
17 Other services (NPISH)	-	-	-	-	-	-	-	-	-	-	-
Total consumption	10368	9124	167623	37856	107729	134561	80262	209331	9070	21910	25721
Imports of goods and services	760	1979	86074	2219	3397	25765	13277	24353	1041	1257	5338
Taxes less subsidies on products	406	174	2702	894	437	5837	3184	8842	-	1626	999
Taxes less subsidies on production	-2975	217	2146	1061	623	8317	1193	2101	-	283	883
Compensation of employees	3202	2986	96367	4640	34005	108704	52499	163798	6091	29130	26328
Gross operating surplus	6667	22722	36556	10164	36206	52342	24136	190173	965	15417	17786
Total output	18428	37201	391468	56835	182396	335526	174551	598599	17168	69623	77054

Product	Non-market - Product			NPISH - Product			Total intermediate demand
	12	13	14	15	16	17	
	Public adminis- tration (non-market) [115]	Education, health and social work (non-market) [116-118]	Other services (non-market) [119-123]	Financial intermed- iation (NPISH) [100-114]	Education, health and social work (NPISH) [116-118]	Other services (NPISH) [119-123]	
1 Agriculture	-	33	-	0	52	6	12115
2 Mining and quarrying	-	3	-	0	1	5	24386
3 Manufacturing	8726	9091	801	19	1007	309	187369
4 Electricity, gas and water supply	838	1106	196	4	109	22	34862
5 Construction	3665	674	310	6	40	23	77319
6 Wholesale & retail trade	3046	5057	242	11	305	93	65518
7 Transport and communication	3618	2663	365	32	497	179	115990
8 Financial intermediation	12636	9024	1587	239	1551	988	315832
9 Public administration	181	-	2	1	12	7	10443
10 Education, health and social work	3502	22342	47	14	986	65	43715
11 Other services	2877	1123	3111	8	390	1069	30921
12 Public administration (non-market)	-	-	-	-	-	-	-
13 Education, health and social work (non-market)	-	-	-	-	-	-	-
14 Other services (non-market)	-	-	-	-	-	-	-
15 Financial intermediation (NPISH)	-	-	-	-	-	-	-
16 Education, health and social work (NPISH)	-	-	-	-	-	-	-
17 Other services (NPISH)	-	-	-	-	-	-	-
Total consumption	39090	51115	6661	333	4948	2766	918469
Imports of goods and services	6410	8978	985	59	270	310	182472
Taxes less subsidies on products	7899	5997	851	5	249	72	40172
Taxes less subsidies on production	-	-	-	-	-	-	13849
Compensation of employees	45762	80048	3605	375	16534	3404	677478
Gross operating surplus	7248	3045	393	389	863	249	425321
Total output	106409	149184	12495	1160	22864	6800	2257761

Product	Final consumption expenditure			Gross capital formation			Exports		Total demand
	Households	General NPISHs	Government	GFCF	Valuables	Changes in inventories	Exports of goods	Exports of services	
	1 Agriculture	4325	-	-	481	-	-84	1509	
2 Mining and quarrying	136	-	-	178	-	103	12169	229	37201
3 Manufacturing	47562	-	-	17122	-997	1778	135511	3123	391468
4 Electricity, gas and water supply	21810	-	-	-	-	-5	98	70	56835
5 Construction	6202	-	-	97154	-	1081	-	640	182396
6 Wholesale & retail trade	231103	-	-	6021	385	-7	21141	11365	335526
7 Transport and communication	38958	-	-	891	15	-14	-	18712	174551
8 Financial intermediation	175669	-	-	32117	2	319	613	74046	598599
9 Public administration	3626	-	-	2036	-	-	-	1063	17168
10 Education, health and social work	23865	-	-	-	-	83	-	1960	69623
11 Other services	36802	-	-	2376	-	-81	3577	3459	77054
12 Public administration (non-market)	-	-	106409	-	-	-	-	-	106409
13 Education, health and social work (non-market)	-	-	149184	-	-	-	-	-	149184
14 Other services (non-market)	-	-	12495	-	-	-	-	-	12495
15 Financial intermediation (NPISH)	-	1160	-	-	-	-	-	-	1160
16 Education, health and social work (NPISH)	-	22864	-	-	-	-	-	-	22864
17 Other services (NPISH)	-	6800	-	-	-	-	-	-	6800
Total consumption	590058	30824	268088	158376	-595	3174	174618	114750	2257761
Imports of goods and services	116284	-	-	39594	218	1442	31207	2424	373641
Taxes less subsidies on products	77799	-	-	11788	-	-144	5784	2012	137410
Total	784140	30824	268088	209758	-377	4472	211608	119186	2768812

The methodology described above cannot be applied to construct a product by product imports use table. This is because anything produced by consuming an import is not regarded as an import. The inputs (imports) are thus conceptually different from the outputs (domestic production). Furthermore, we have no breakdown of imports by foreign producer, only by product. The 2005

product by product imports use table has therefore been constructed by applying *iterative proportional scaling* (also known as *raking* and *rAs*) to the IOT as follows:

- The intermediate demand parts of the rows are scaled to sum to the intermediate demand totals for the product by industry imports use table.
- The intermediate demand parts of the columns are scaled to sum to the value of imports in the primary inputs section of the IOT.
- The above scalings are repeated until convergence.
- The final demand section of the product by industry imports use table is attached to the right of the resulting table.

Table 4 shows the summary 2005 product by product imports use table.

**Table 4: Summary Imports Use table (product by product), 2005**

£ million											
Product	Intermediate consumption by products										
	1	2	3	4	5	6	7	8	9	10	11
	Agriculture [1-3]	Mining and quarrying [4-7]	Manufacturing [8-84]	Electricity, Gas and water supply [85-87]	Construction [88]	Wholesale and retail trade [89-92]	Transport and communication [93-99]	Financial intermediation [100-114]	Public administration [115]	Education, health and social work [116-118]	Other services [119-123]
1 Agriculture	87	0	940	0	9	231	26	11	-	5	10
2 Mining and quarrying	0	1047	12047	1881	302	178	42	455	-	1	7
3 Manufacturing	576	541	62399	220	2274	15733	8312	6768	976	885	2409
4 Electricity, gas and water supply	2	2	246	27	1	32	9	23	0	1	4
5 Construction	2	9	57	2	221	30	28	129	1	1	5
6 Wholesale & retail trade	8	21	1023	7	37	1189	160	644	8	29	137
7 Transport and communication	19	87	1952	6	31	2629	2273	1736	7	34	364
8 Financial intermediation	55	263	7013	73	515	5560	2285	14001	42	244	1384
9 Public administration	0	0	9	0	1	1	5	27	0	0	1
10 Education, health and social work	4	1	116	2	3	52	53	169	3	36	73
11 Other services	8	7	273	2	4	128	84	390	5	22	945
12 Public administration (non-market)	-	-	-	-	-	-	-	-	-	-	-
13 Education, health and social work (non-market)	-	-	-	-	-	-	-	-	-	-	-
14 Other services (non-market)	-	-	-	-	-	-	-	-	-	-	-
15 Financial intermediation (NPISH)	-	-	-	-	-	-	-	-	-	-	-
16 Education, health and social work (NPISH)	-	-	-	-	-	-	-	-	-	-	-
17 Other services (NPISH)	-	-	-	-	-	-	-	-	-	-	-
<b>Total imports</b>	<b>760</b>	<b>1979</b>	<b>86074</b>	<b>2219</b>	<b>3397</b>	<b>25765</b>	<b>13277</b>	<b>24353</b>	<b>1041</b>	<b>1257</b>	<b>5338</b>

Product	Non-market - Product			NPISH - Product			Total intermediate demand
	12	13	14	15	16	17	
	Public administration (non-market) [115]	Education, health and social work (non-market) [116-118]	Other services (non-market) [119-123]	Financial intermediation (NPISH) [100-114]	Education, health and social work (NPISH) [116-118]	Other services (NPISH) [119-123]	
1 Agriculture	-	1	-	0	3	0	1323
2 Mining and quarrying	-	3	-	0	0	0	15864
3 Manufacturing	5313	8554	449	18	154	222	115804
4 Electricity, gas and water supply	2	1	2	0	0	0	354
5 Construction	21	1	5	0	0	0	512
6 Wholesale & retail trade	205	64	18	2	13	6	3570
7 Transport and communication	114	21	26	2	11	7	9318
8 Financial intermediation	616	249	223	35	74	38	32670
9 Public administration	0	-	0	0	0	0	44
10 Education, health and social work	55	73	2	1	10	1	654
11 Other services	83	12	260	0	5	33	2260
12 Public administration (non-market)	-	-	-	-	-	-	-
13 Education, health and social work (non-market)	-	-	-	-	-	-	-
14 Other services (non-market)	-	-	-	-	-	-	-
15 Financial intermediation (NPISH)	-	-	-	-	-	-	-
16 Education, health and social work (NPISH)	-	-	-	-	-	-	-
17 Other services (NPISH)	-	-	-	-	-	-	-
<b>Total imports</b>	<b>6410</b>	<b>8978</b>	<b>985</b>	<b>59</b>	<b>270</b>	<b>310</b>	<b>182472</b>

Product	Final consumption expenditure			Gross capital formation			Exports		Total demand
	Households	NPISHs	General	GFCF	Valuables	Changes in inventories	Exports of goods	Exports of services	
			government						
1 Agriculture	5723	-	-	136	-	-25	-	-	7157
2 Mining and quarrying	28	-	-	213	-	187	3976	-	20367
3 Manufacturing	82937	-	-	38378	235	1279	27231	-	265863
4 Electricity, gas and water supply	121	-	-	-	-	-	-	-	475
5 Construction	124	-	-	-	-	-	-	-	636
6 Wholesale & retail trade	10416	-	-	-	-	-	-	-	13986
7 Transport and communication	8578	-	-	180	-15	-	-	2424	20485
8 Financial intermediation	1927	-	-	577	-2	2	-	-	35173
9 Public administration	-	-	-	-	-	-	-	-	44
10 Education, health and social work	1347	-	-	-	-	-	-	-	2001
11 Other services	5084	-	-	110	-	-	-	-	7454
12 Public administration (non-market)	-	-	-	-	-	-	-	-	-
13 Education, health and social work (non-market)	-	-	-	-	-	-	-	-	-
14 Other services (non-market)	-	-	-	-	-	-	-	-	-
15 Financial intermediation (NPISH)	-	-	-	-	-	-	-	-	-
16 Education, health and social work (NPISH)	-	-	-	-	-	-	-	-	-
17 Other services (NPISH)	-	-	-	-	-	-	-	-	-
<b>Total imports</b>	<b>116284</b>	<b>-</b>	<b>-</b>	<b>39594</b>	<b>218</b>	<b>1442</b>	<b>31207</b>	<b>2424</b>	<b>373641</b>

## Matrix of Coefficients

The matrix of coefficients is an important tool in the study of the underlying structure of the economy. It can be calculated by dividing each column of the intermediate demand and primary inputs part of the IOT by the column total. It has the following properties:

- Apart from those primary inputs with negative entries in the IOT, the coefficients take values between 0 and 1 (provided that GVA is positive). They describe the distribution of inputs by monetary value for each product IOG.

- It covers intermediate and primary inputs.
- Each column sums to 1.
- Changes in the coefficients indicate changes in the structure of the economy. They assist in the study of relative price changes, capital/labour intensity, developments in technology, product substitution and import substitution, for example.

Although the matrix of coefficients is conceptually derived from the IOT, applying matrix algebra makes it more convenient to calculate the matrix of coefficients first and then to construct the IOT. The intermediate demand part of the matrix is traditionally denoted by the letter **A**. The 2005 summary matrix of coefficients is shown in table 5.

**Table 5: Summary matrix of coefficients (product by product), 2005**

Product	Intermediate consumption by products										
	1	2	3	4	5	6	7	8	9	10	11
	Agriculture	Mining and quarrying	Manufacturing	Electricity, Gas and water supply	Construction	Wholesale and retail trade	Transport and communication	Financial intermediation	Public administration	Education, health and social work	Other services
	[1-3]	[4-7]	[8-84]	[85-87]	[88]	[89-92]	[93-99]	[100-114]	[115]	[116-118]	[119-123]
1 Agriculture	0.074	0.000	0.022	0.000	0.001	0.005	0.000	0.000	-	0.002	0.001
2 Mining and quarrying	0.000	0.060	0.010	0.258	0.014	0.001	0.001	0.001	-	0.000	0.000
3 Manufacturing	0.234	0.034	0.200	0.045	0.124	0.073	0.067	0.021	0.148	0.057	0.043
4 Electricity, gas and water supply	0.022	0.019	0.023	0.260	0.003	0.008	0.006	0.004	0.011	0.006	0.005
5 Construction	0.012	0.018	0.003	0.016	0.288	0.006	0.012	0.020	0.045	0.003	0.003
6 Wholesale & retail trade	0.073	0.015	0.062	0.011	0.026	0.035	0.022	0.013	0.012	0.015	0.012
7 Transport and communication	0.021	0.030	0.031	0.008	0.008	0.097	0.175	0.043	0.046	0.026	0.025
8 Financial intermediation	0.105	0.065	0.068	0.062	0.123	0.167	0.151	0.220	0.182	0.086	0.125
9 Public administration	0.001	0.001	0.001	0.001	0.002	0.001	0.011	0.012	0.002	0.001	0.001
10 Education, health and social work	0.008	0.001	0.003	0.004	0.001	0.003	0.006	0.008	0.043	0.094	0.010
11 Other services	0.011	0.002	0.006	0.003	0.001	0.006	0.008	0.008	0.038	0.024	0.108
12 Public administration (non-market)	-	-	-	-	-	-	-	-	-	-	-
13 Education, health and social work (non-market)	-	-	-	-	-	-	-	-	-	-	-
14 Other services (non-market)	-	-	-	-	-	-	-	-	-	-	-
15 Financial intermediation (NPISH)	-	-	-	-	-	-	-	-	-	-	-
16 Education, health and social work (NPISH)	-	-	-	-	-	-	-	-	-	-	-
17 Other services (NPISH)	-	-	-	-	-	-	-	-	-	-	-
Total consumption	0.563	0.245	0.428	0.666	0.591	0.401	0.460	0.350	0.528	0.315	0.334
Imports of goods and services	0.041	0.053	0.220	0.039	0.019	0.077	0.076	0.041	0.061	0.018	0.069
Taxes less subsidies on products	0.022	0.005	0.007	0.016	0.002	0.017	0.018	0.015	-	0.023	0.013
Taxes less subsidies on production	-0.161	0.006	0.005	0.019	0.003	0.025	0.007	0.004	-	0.004	0.011
Compensation of employees	0.174	0.080	0.246	0.082	0.186	0.324	0.301	0.274	0.355	0.418	0.342
Gross operating surplus	0.362	0.611	0.093	0.179	0.199	0.156	0.138	0.318	0.056	0.221	0.231
Total output	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Product	Non-market - Product			NPISH - Product		
	12	13	14	15	16	17
	Public administration (non-market)	Education, health and social work (non-market)	Other services (non-market)	Financial intermediation (NPISH)	Education, health and social work (NPISH)	Other services (NPISH)
	[115]	[116-118]	[119-123]	[100-114]	[116-118]	[119-123]
1 Agriculture	-	0.000	-	0.000	0.002	0.001
2 Mining and quarrying	-	0.000	-	0.000	0.000	0.001
3 Manufacturing	0.082	0.061	0.064	0.016	0.044	0.045
4 Electricity, gas and water supply	0.008	0.007	0.016	0.003	0.005	0.003
5 Construction	0.034	0.005	0.025	0.005	0.002	0.003
6 Wholesale & retail trade	0.029	0.034	0.019	0.009	0.013	0.014
7 Transport and communication	0.034	0.018	0.029	0.027	0.022	0.026
8 Financial intermediation	0.119	0.060	0.127	0.206	0.068	0.145
9 Public administration	0.002	-	0.000	0.000	0.001	0.001
10 Education, health and social work	0.033	0.150	0.004	0.012	0.043	0.010
11 Other services	0.027	0.008	0.249	0.007	0.017	0.157
12 Public administration (non-market)	-	-	-	-	-	-
13 Education, health and social work (non-market)	-	-	-	-	-	-
14 Other services (non-market)	-	-	-	-	-	-
15 Financial intermediation (NPISH)	-	-	-	-	-	-
16 Education, health and social work (NPISH)	-	-	-	-	-	-
17 Other services (NPISH)	-	-	-	-	-	-
Total consumption	0.367	0.343	0.533	0.287	0.216	0.407
Imports of goods and services	0.060	0.060	0.079	0.050	0.012	0.046
Taxes less subsidies on products	0.074	0.040	0.068	0.004	0.011	0.011
Taxes less subsidies on production	-	-	-	-	-	-
Compensation of employees	0.430	0.537	0.289	0.323	0.723	0.501
Gross operating surplus	0.068	0.020	0.031	0.336	0.038	0.037
Total output	1.000	1.000	1.000	1.000	1.000	1.000

## Leontief Inverse

Whereas the matrix of coefficients helps to analyse direct relationships within the economy, the Leontief Inverse takes indirect relationships into account. In order to interpret the Leontief Inverse, it is useful to consider its derivation in terms of the identities in the tables. The main identity across the rows of the IOT is:

$$\text{Total demand (q)} = \text{Intermediate demand} + \text{Final demand (f)} \quad (1)$$

**Intermediate demand** is the demand for products which are used-up or changed by producers. **Final demand** consists of final consumption expenditure (of households, NPISHs and general government), gross capital formation (gross fixed capital formation, acquisition less disposal of valuables and changes in inventories) and exports of goods and services.

We can use the intermediate demand part of the matrix of coefficients to rewrite equation (1) as:

$$\mathbf{q} = \mathbf{Aq} + \mathbf{f} \quad (2)$$

After a little matrix algebra (see Annex C), equation (2) can be re-expressed as:

$$\mathbf{q} = (\mathbf{I} - \mathbf{A})^{-1} \mathbf{f} \quad (3)$$

where  $\mathbf{I}$  is the identity matrix and  $(\mathbf{I} - \mathbf{A})^{-1}$  is the Leontief Inverse. The Leontief Inverse thus describes the relationship between total demand and final demand. Table 6 shows a summary version of the Leontief Inverse for 2005. The right hand side of equation (3) may be decomposed as follows:

$$(\mathbf{I} - \mathbf{A})^{-1} \mathbf{f} = \mathbf{f} + \mathbf{Af} + \mathbf{A}^2\mathbf{f} + \mathbf{A}^3\mathbf{f} + \mathbf{A}^4\mathbf{f} + \dots \quad (4)$$

The first term in this decomposition is final demand. The second term (the first order effects) gives the intermediate demand directly generated from the final demand. This direct intermediate demand generates further intermediate demand represented by the third term (the second order effects). This in turn generates even further intermediate demand represented by the fourth term, and so on. In other words, to produce  $\mathbf{f}$  requires the consumption of  $\mathbf{Af}$ , which requires the further consumption of  $\mathbf{A}^2\mathbf{f}$ , which in turn requires the further consumption of  $\mathbf{A}^3\mathbf{f}$ ... Equation (4) thus analyses the total demand required to satisfy final demand  $\mathbf{f}$ . In this way, the Leontief Inverse allows the analyst to study the impact of changes in final demand (for example, due to government programmes) on the economy as a whole. The linear nature of the model described by equation (4) makes it very flexible but unable to take into account factors such as economies of scale. The model is therefore an approximation.

**Table 6: Summary Leontief Inverse (product by product), 2005**

Product	Intermediate consumption by products										
	1	2	3	4	5	6	7	8	9	10	11
	Agriculture	Mining and quarrying	Manufacturing	Electricity, Gas and water supply	Construction	Wholesale and retail trade	Transport and communication	Financial intermediation	Public administration	Education, health and social work	Other services
	[1-3]	[4-7]	[8-84]	[85-87]	[88]	[89-92]	[93-99]	[100-114]	[115]	[116-118]	[119-123]
1 Agriculture	1.089	0.002	0.031	0.003	0.008	0.009	0.004	0.002	0.006	0.005	0.003
2 Mining and quarrying	0.018	1.073	0.026	0.376	0.028	0.008	0.009	0.006	0.011	0.005	0.005
3 Manufacturing	0.352	0.064	1.284	0.114	0.242	0.125	0.127	0.056	0.228	0.096	0.078
4 Electricity, gas and water supply	0.048	0.031	0.045	1.366	0.016	0.019	0.017	0.010	0.027	0.014	0.012
5 Construction	0.031	0.033	0.015	0.047	1.416	0.020	0.031	0.040	0.077	0.011	0.013
6 Wholesale & retail trade	0.112	0.025	0.090	0.034	0.059	1.053	0.042	0.025	0.037	0.028	0.024
7 Transport and communication	0.069	0.052	0.070	0.045	0.047	0.144	1.239	0.076	0.090	0.052	0.052
8 Financial intermediation	0.231	0.121	0.160	0.177	0.274	0.275	0.275	1.323	0.311	0.157	0.209
9 Public administration	0.005	0.003	0.004	0.004	0.007	0.006	0.017	0.016	1.008	0.004	0.004
10 Education, health and social work	0.015	0.003	0.007	0.008	0.006	0.008	0.013	0.014	0.053	1.107	0.015
11 Other services	0.020	0.005	0.013	0.009	0.008	0.013	0.016	0.015	0.050	0.033	1.125
12 Public administration (non-market)	-	-	-	-	-	-	-	-	-	-	-
13 Education, health and social work (non-market)	-	-	-	-	-	-	-	-	-	-	-
14 Other services (non-market)	-	-	-	-	-	-	-	-	-	-	-
15 Financial intermediation (NPISH)	-	-	-	-	-	-	-	-	-	-	-
16 Education, health and social work (NPISH)	-	-	-	-	-	-	-	-	-	-	-
17 Other services (NPISH)	-	-	-	-	-	-	-	-	-	-	-
<b>Total</b>	<b>1.989</b>	<b>1.412</b>	<b>1.745</b>	<b>2.185</b>	<b>2.112</b>	<b>1.679</b>	<b>1.790</b>	<b>1.582</b>	<b>1.898</b>	<b>1.513</b>	<b>1.541</b>

Product	Non-market - Product			NPISH - Product		
	12	13	14	15	16	17
	Public administration (non-market)	Education, health and social work (non-market)	Other services (non-market)	Financial intermediation (NPISH)	Education, health and social work (NPISH)	Other services (NPISH)
	[115]	[116-118]	[119-123]	[100-114]	[116-118]	[119-123]
1 Agriculture	0.004	0.003	0.003	0.001	0.004	0.003
2 Mining and quarrying	0.008	0.006	0.011	0.003	0.004	0.005
3 Manufacturing	0.135	0.105	0.123	0.041	0.072	0.087
4 Electricity, gas and water supply	0.018	0.017	0.030	0.008	0.011	0.011
5 Construction	0.058	0.013	0.047	0.018	0.008	0.015
6 Wholesale & retail trade	0.046	0.048	0.039	0.018	0.023	0.028
7 Transport and communication	0.066	0.045	0.068	0.053	0.041	0.058
8 Financial intermediation	0.210	0.132	0.254	0.290	0.119	0.247
9 Public administration	0.005	0.002	0.004	0.005	0.003	0.005
10 Education, health and social work	0.040	0.168	0.011	0.017	0.050	0.016
11 Other services	0.036	0.016	0.284	0.012	0.023	0.181
12 Public administration (non-market)	1.000	-	-	-	-	-
13 Education, health and social work (non-market)	-	1.000	-	-	-	-
14 Other services (non-market)	-	-	1.000	-	-	-
15 Financial intermediation (NPISH)	-	-	-	1.000	-	-
16 Education, health and social work (NPISH)	-	-	-	-	1.000	-
17 Other services (NPISH)	-	-	-	-	-	1.000
<b>Total</b>	<b>1.625</b>	<b>1.555</b>	<b>1.874</b>	<b>1.467</b>	<b>1.357</b>	<b>1.655</b>

## Multiplier Analysis

The Leontief Inverse provides the central tool for multiplier analysis, which studies the effect of changes in final demand on output and related aspects of the economy. These effects have three different economic drivers:

- **Direct:** This is the immediate effect caused directly by the change in final demand.
- **Indirect:** This is the subsequent effect caused by the consequent changes in intermediate demand.
- **Induced:** This is the effect attributable to the ensuing change in compensation of employees and other incomes, which may cause further spending and hence further changes in final demand.

**Type I multipliers** cover direct and indirect effects only, and therefore underestimate the effect on the economy. **Type II multipliers** cover induced effects as well, under the implicit assumption that final consumers do not change their final consumption patterns in response to changes in income. Because this assumption is very unrealistic and because of the lack of consistent employment numbers, only Type I multipliers have been calculated for this publication.

Different multipliers measure the effect on different policy targets:

- **Output multipliers** measure the effect on total economic output caused by a one unit change in the final demand of a specific product. They are calculated as the column totals of the Leontief Inverse.
- We have not produced employment multipliers because we do not have employment numbers coherent with the balanced compensation of employees data for 2005. However, we have produced **employment cost multipliers** which measure the relative effect on total compensation of employees caused by a one unit change in the final demand of a specific product. They are calculated as:
 
$$\mathbf{e} (\mathbf{I}-\mathbf{A})^{-1} / \mathbf{e} \quad (5)$$
 where  $\mathbf{e}$  is the row of the matrix of coefficients corresponding to compensation of employees, and  $/$  denotes to element-wise division.
- It is possible to produce other multipliers by substituting  $\mathbf{e}$  in equation (5) with appropriate coefficients (for examples see Scottish Government, 2011). It may be noted that output multipliers can be calculated in this way by substituting a row vector of ones for  $\mathbf{e}$ .

## Primary Input Content of Final Demand

The Leontief Inverse can also be used to analyse the content of final demand in terms of primary inputs. These are inputs to the production process which are not the outputs of other domestic producers: imports; taxes less subsidies on products and production; compensation of employees; and gross operating surplus. Indirectly, all final demand is ultimately satisfied by primary inputs. Thus the sum of all primary inputs is equal to the sum of all final demand. This holds at both basic prices and purchasers' prices. The formula for this analysis is:

$$\mathbf{P} (\mathbf{I}-\mathbf{A})^{-1} \mathbf{F}$$

where  $\mathbf{P}$  is that part of the matrix of coefficients corresponding to primary inputs, and  $\mathbf{F}$  is the matrix of final demand at basic prices.

Table 7 shows the decomposition of final demand in terms of primary inputs at basic prices (ie excluding taxes less subsidies on products and imports directly consumed by the components of final demand) for 2005, expressed as percentages of each final demand component. Table 8 shows the same decomposition expressed in monetary values. Rows of totals relating to taxes less subsidies on products and imports directly consumed by the components of final demand have been appended to show totals in purchasers' prices. From this table GDP at market prices (£1,254,058m) can be derived by deducting the direct (£191,169m) and indirect (£182,472m) contributions of imports to final demand from the grand total (£1,627,699m).

The primary input content of final demand can also be used to partition GVA into **direct** and **indirect** effects. The direct effects show how much GVA is directly attributable to the different components of final demand. The formula for their calculation is:

$$\mathbf{i} \mathbf{F}$$

where  $\mathbf{i}$  is the row vector of column sums of that part of the matrix of coefficients corresponding to the income measure of GVA (compensation of employees, gross operating surplus, taxes less subsidies on production). The contribution of each product IOG to direct GVA is obtained by diagonalising the vector  $\mathbf{i}$  in the formula.

The indirect effects show how much GVA is attributable to all the subsequent demand following the direct effects. They are calculated by subtracting the direct effects from the total effects. The formula for the total effects is:

$$\mathbf{i} (\mathbf{I}-\mathbf{A})^{-1} \mathbf{F}$$

The contribution of each product IOG to total GVA is obtained by diagonalising the vector  $\mathbf{i} (\mathbf{I}-\mathbf{A})^{-1}$  in the formula.

Figure 5 depicts the percentages of direct and indirect GVA by component of final demand.

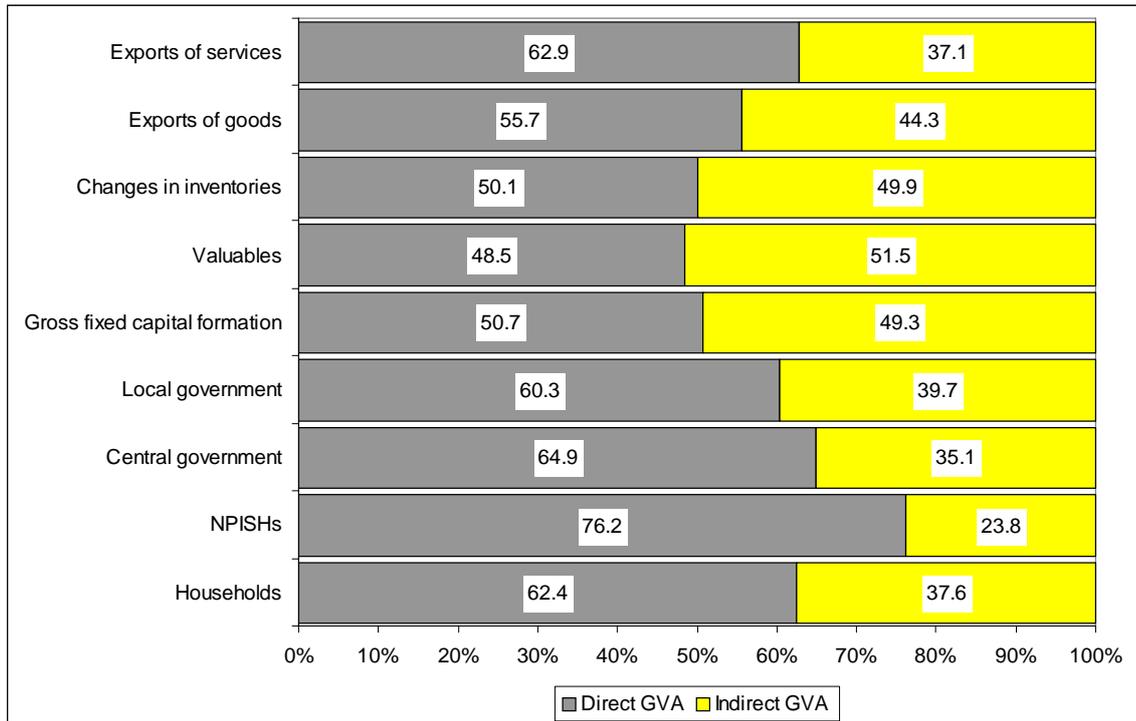
**Table 7: Primary input content of final demand at basic prices (as a percentage), 2005**

Primary input	Final consumption expenditure				Gross capital formation			Exports		Total final demand	percentages
	Households	NPISHs	Central	Local	GFCF	Valuables	Changes in inventories	Exports of goods	Exports of services		
			government	government							
Imports of goods and services	11.2	5.6	12.7	7.8	12.4	50.3	27.5	29.3	12.7	13.6	
Taxes less subsidies on products	2.6	1.6	6.2	6.4	1.1	1.1	1.3	1.5	2.6	3.0	
Taxes less subsidies on production	1.4	0.2	0.4	0.4	1.1	-1.3	1.7	0.9	1.0	1.0	
Compensation of employees	43.8	79.7	64.9	70.6	49.6	36.1	44.4	41.9	53.5	50.6	
Gross operating surplus	41.0	13.0	15.9	14.7	35.8	13.8	25.0	26.4	30.2	31.8	
<b>Total primary input</b>	<b>100.0</b>	<b>100.0</b>	<b>100.0</b>	<b>100.0</b>	<b>100.0</b>	<b>100.0</b>	<b>100.0</b>	<b>100.0</b>	<b>100.0</b>	<b>100.0</b>	

**Table 8: Primary input content of final demand at purchasers' prices, 2005**

Primary input	Final consumption expenditure				Gross capital formation			Exports		Total final demand	£ million
	Households	NPISHs	Central	Local	GFCF	Valuables	Changes in inventories	Exports of goods	Exports of services		
			government	government							
Imports of goods and services	65906	1715	20482	8372	19618	-299	873	51243	14563	182472	
Taxes less subsidies on products	15550	481	9967	6877	1744	-6	42	2587	2931	40172	
Taxes less subsidies on production	8383	62	565	427	1699	7	55	1523	1127	13849	
Compensation of employees	258436	24565	104645	75338	78622	-215	1410	73240	61437	677478	
Gross operating surplus	241783	4001	25670	15745	56693	-82	794	46025	34692	425321	
<b>Total primary input at basic prices</b>	<b>590058</b>	<b>30824</b>	<b>161329</b>	<b>106759</b>	<b>158376</b>	<b>-595</b>	<b>3174</b>	<b>174618</b>	<b>114750</b>	<b>1339293</b>	
Imports in final demand	116284	-	-	-	39594	218	1442	31207	2424	191169	
Taxes less subsidies in final demand	77799	-	-	-	11788	-	-144	5784	2012	97238	
<b>Total primary input at purchasers' prices</b>	<b>784140</b>	<b>30824</b>	<b>161329</b>	<b>106759</b>	<b>209758</b>	<b>-377</b>	<b>4472</b>	<b>211608</b>	<b>119186</b>	<b>1627699</b>	

**Figure 5: Composition of final demand in terms of direct and indirect GVA (%), 2005**



## Industrial Analysis in terms of Primary Inputs

So far, the analysis in this publication has been based on product by product tables. Similar analyses can also be produced based on industry by industry tables. The methodology to produce these can be found in Eurostat (2008, ch11) and Scottish Government (2011). The compilation of industry by industry tables presents certain conceptual issues not encountered in product by product tables. Most notably:

- Assumptions are required relating to the structure of demand of industry outputs. However, demand is by nature product-driven rather than industry-driven.
- The input-output model is built on an assumption of homogeneity within IOGs. However, industry IOGs are mostly less homogeneous than their corresponding product IOGs.

Eurostat (2008, p301) explains why ESA 1995 (Eurostat, 1996) favours product by product tables for economic analysis.

An alternative approach is to use product by product tables for an industry-based primary input analysis as follows:

- The **Market Share Mix hypothesis** (MSM) is introduced. This uses the assumption that the contribution (market share) of each industry to the supply of a particular product remains a constant proportion of that product's supply.
- The product by product Leontief Inverse is used to determine the product output generated by each component of final demand, as was done for the analysis of primary inputs, above.

- The MSM is used to obtain the equivalent industry output.
- The product by industry domestic use table at basic prices is used to generate a new (product by industry) matrix of coefficients for each industry.
- The amount of primary inputs generated by each component of final demand can then be calculated.

Tables 9 and 10 show the result of this industrial analysis for 2005 as percentages and monetary values, respectively. The matrix algebra for this analysis is detailed in Annex D.

**Table 9: Industrial analysis of primary inputs (as a percentage), 2005**

Primary input	Final consumption expenditure				Gross capital formation			Exports		Total final demand
	Households	NPISHs	Central	Local	GFCF	Valuables	Changes in inventories	Exports of goods	Exports of services	
			government	government						
Imports of goods and services	36.2	0.9	11.2	4.6	10.7	-0.2	0.5	28.0	7.9	100.0
Taxes less subsidies on products	38.6	1.2	24.8	17.1	4.4	0.0	0.1	6.4	7.3	100.0
Taxes less subsidies on production	60.0	0.5	4.1	3.1	12.4	0.1	0.4	11.0	8.4	100.0
Compensation of employees	38.2	3.6	15.4	11.1	11.6	0.0	0.2	10.8	9.0	100.0
Gross operating surplus	56.7	0.9	6.0	3.7	13.4	0.0	0.2	10.8	8.2	100.0

**Table 10: Industrial analysis of primary inputs, 2005**

Primary input	Final consumption expenditure				Gross capital formation			Exports		Total final demand
	Households	NPISHs	Central	Local	GFCF	Valuables	Changes in inventories	Exports of goods	Exports of services	
			government	government						
Imports of goods and services	66104	1712	20493	8386	19610	-300	873	51096	14499	182472
Taxes less subsidies on products	15524	481	9969	6878	1769	-6	42	2584	2931	40172
Taxes less subsidies on production	8315	65	570	425	1718	7	55	1524	1170	13849
Compensation of employees	258869	24551	104616	75324	78444	-214	1409	73229	61250	677478
Gross operating surplus	241268	4008	25667	15762	56896	-80	796	46136	34868	425321
Total	590080	30816	161315	106775	158437	-594	3176	174569	114719	1339293

## Summary of changes in the 2005 Input-Output Analytical Tables

A number of changes have been introduced for the 2005 IOATs. The main ones are summarised below:

- Consistent with Standard Industrial Classification 2003 (SIC (03)) for industries and with the Classification of Products by Activity 2002 (CPA (02)) for products.
- Use of detailed imports data from Her Majesty's Revenue and Customs supplied through the provisions of the Statistics and Registration Service Act 2007.
- There have been a number of methodological improvements in the SUTs from which the IOATs have been constructed, most notably the treatment of Financial Intermediation Services Indirectly Measured (FISIM).
- Constructed from SUTs compiled using a different balancing system.
- Complete review of hybrid technology assumption matrix.
- Compiled in accordance with the Code of Practice for Official Statistics (UK Statistics Authority, 2009).

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## Annex A: Definition of Input-Output Groups

IOG code	non-market split?	NPISH split?	Product and industry description	CPA (02) / SIC (03) Definitions	NACE Rev 1.1 industrial classifications		
					Division	Subsection	Section
1			Agriculture	01	01	A	A
2			Forestry	02	02	A	A
3			Fishing	05	05	B	B
4			Coal extraction	10	10	CA (pt)	C
5			Oil and gas extraction	11	11		
6-7			Metal ores extraction, Other mining and quarrying	12 to 14	12 to 14	CA (pt), CB	
8			Meat processing	15.1	15	DA	D
9			Fish and fruit processing	15.2, 15.3			
10			Oils and fats processing	15.4			
11			Dairy products	15.5			
12			Grain milling and starch	15.6			
13			Animal feed	15.7			
14			Bread, biscuits, etc	15.81, 15.82			
15			Sugar	15.83			
16			Confectionery	15.84			
17			Other food products	15.85 to 15.89			
18			Alcoholic beverages	15.91 to 15.97			
19			Soft drinks & mineral waters	15.98			
20			Tobacco products	16	16		
21-23			Textile fibres, Textile weaving, Textile finishing	17.1 to 17.3	17	DB	D
24-27			Made-up textiles, Carpets and rugs, Other textiles, Knitted goods	17.4 to 17.7			
28			Wearing apparel & fur products	18	18		
29-30			Leather goods, Footwear	19	19	DC	
31			Wood and wood products	20	20	DD	
32			Pulp, paper and paperboard	21.1	21	DE	D
33			Paper and paperboard products	21.2			
34			Printing and publishing	22	22		
35			Coke ovens, refined petroleum & nuclear fuel	23	23	DF	
36			Industrial gases and dyes	24.11, 24.12	24	DG	D
37-38			Inorganic chemicals, Organic chemicals	24.13, 24.14			
39-41			Fertilisers, Plastics & Synthetic resins etc, Pesticides	24.15 to 24.2			
42			Paints, varnishes, printing ink etc	24.3			
43			Pharmaceuticals	24.4	25	DH	D
44			Soap and toilet preparations	24.5			
45-46			Other Chemical products, Man-made fibres	24.6, 24.7			
47			Rubber products	25.1	25	DH	D
48			Plastic products	25.2			
49			Glass and glass products	26.1	26	DI	D
50			Ceramic goods	26.2, 26.3			
51-52			Structural clay products, Cement, lime and plaster	26.4, 26.5	26	DI	D
53			Articles of concrete, stone etc	26.6 to 26.8			
54-56			Iron and steel, Non-ferrous metals, Metal castings	27	27		
57			Structural metal products	28.1	28	DJ	D
58			Metal boilers & radiators	28.2, 28.3			
59			Metal forging, pressing, etc	28.4, 28.5			
60			Cutlery, tools etc	28.6			
61			Other Metal products	28.7	29	DK	D
62			Mechanical power equipment	29.1			
63			General purpose machinery	29.2			
64			Agricultural machinery	29.3			
65			Machine tools	29.4			
66			Special purpose machinery	29.5			
67			Weapons and ammunition	29.6			
68			Domestic appliances not elsewhere classified	29.7			
69			Office machinery & computers	30			
70-71			Electric motors and generators etc, Insulated wire and cable	31.1 to 31.3			
72			Electrical equipment not elsewhere classified	31.4 to 31.6	31	DL	D
73			Electronic components	32.1			
74			Transmitters for TV, radio and phone	32.2			
75			Receivers for TV and radio	32.3	32	DL	D
76			Medical and precision instruments	33			
77			Motor vehicles	34	34		
78			Shipbuilding and repair	35.1	35	DM	D
79			Other transport equipment	35.2, 35.4, 35.5			
80			Aircraft and spacecraft	35.3			
81			Furniture	36.1	36 & 37	DN	D
82			Jewellery & related products	36.2, 36.3			
83			Sports goods and toys	36.4, 36.5			
84			Miscellaneous manufacturing not elsewhere classified, recycling	36.6, 37			

IOG code	non- market	NPISH split?	Product and industry description	CPA (02) / SIC (03)	NACE Rev 1.1				
					Division	Subsection	Section		
85			Electricity production & distribution	40.1	40	E	E		
86			Gas distribution	40.2, 40.3					
87			Water supply	41	41				
88			Construction	45	45	F	F		
89			Motor vehicle distribution & repair, fuel	50	50	G	G		
90			Wholesale distribution	51	51				
91			Retail distribution	52	52				
92			Hotels, catering, pubs etc	55	55	H	H		
93			Railway transport	60.1	60				
94			Other land transport	60.2, 60.3					
95			Water transport	61	61	I	I		
96			Air Transport	62	62				
97			Ancillary Transport services	63	63				
98			Postal and courier services	64.1	64				
99			Telecommunications	64.2					
100			Banking and finance	65	65				
101		yes	Insurance and pension funds	66	66	J	J		
102			Auxiliary financial services	67	67				
103			Owning and dealing in real estate	70.1, 70.2 (pt)	70				
104			Letting of dwellings	70.2 (pt)					
105			Estate agent activities	70.3					
106			Renting of machinery etc	71				71	
107			Computer services	72	72	K	K		
108		yes	Research and development	73	73				
109			Legal activities	74.11	74				
110			Accountancy services	74.12					
111			Market research, management consultancy	74.13 to 74.15	74				
112			Architectural activities & Tech. Consult	74.2, 74.3					
113			Advertising	74.4					
114		yes	Other business services	74.5 to 74.8					
115	yes		Public administration & defence	75	75	L	L		
116	yes	yes	Education	80	80	M	M		
117	yes	yes	Health and veterinary services	85.1, 85.2	85	N	N		
118	yes	yes	Social work activities	85.3					
119	yes		Sewage and Sanitary services	90	90	O	O		
120		yes	Membership organisations not elsewhere classified	91	91				
121	yes	yes	Recreational services	92	92				
122		yes	Other service activities	93	93				
123			Private Households with employed persons	95	95	P	P		

## Annex B: Matrix Algebra for the Input-Output Table

The following notation is used:

**M** is the supply table excluding the columns for imports, margins and taxes less subsidies

**q** is the vector of domestic supply of product IOGs at basic prices

**U** is the domestic use table at basic prices, including final demand and primary inputs

**H** is the hybrid technology assumption matrix

Let:

$$\mathbf{M}_1 = \mathbf{M} \# \mathbf{H} \quad \text{where } \# \text{ denotes element-wise multiplication}$$

$$\mathbf{M}_2 = \mathbf{M} - \mathbf{M}_1$$

$$\mathbf{C}_1 = \mathbf{M}_1 (\text{diag}(\mathbf{1}' \mathbf{M}_1))^{-1} \quad \text{where } \mathbf{1} \text{ is a column vector of ones}$$

$$\mathbf{D}_2 = \mathbf{M}_2' (\text{diag}(\mathbf{q}))^{-1}$$

$$\mathbf{R} = \mathbf{C}_1^{-1} (\mathbf{I} - \text{diag}(\mathbf{D}_2' \mathbf{1})) + \mathbf{D}_2$$

$$\mathbf{B} = \mathbf{U} \text{diag}(\mathbf{U} \mathbf{1})^{-1}$$

Now let **B\*** be matrix **B** without the columns corresponding to final demand:

$$\mathbf{A}^* = \mathbf{B}^* \mathbf{R}$$

where **A\*** is the matrix of coefficients, including the rows corresponding to primary inputs.

Finally,

$$\mathbf{S} = \mathbf{A}^* \# (\mathbf{1} \mathbf{1}' \mathbf{M}')$$

where **S** is the input-output table excluding the columns corresponding to final demand (the components of final demand may be copied from the domestic use table at basic prices).

## Annex C: Derivation of the Leontief Inverse

We recall equation (2):

$$\mathbf{q} = \mathbf{A}\mathbf{q} + \mathbf{f}$$

where  $\mathbf{q}$  is the vector of total demand,

$\mathbf{f}$  is the vector of final demand (summed over components), and

$\mathbf{A}$  is the main body of the matrix of coefficients (the intermediate demand part only).

Subtracting  $\mathbf{A}\mathbf{q}$  from both sides gives:

$$\mathbf{q} - \mathbf{A}\mathbf{q} = \mathbf{f}$$

Which may be rewritten as:

$$(\mathbf{I} - \mathbf{A}) \mathbf{q} = \mathbf{f}$$

where  $\mathbf{I}$  is the identity matrix.

$(\mathbf{I} - \mathbf{A})$  is called the *Leontief matrix*.

Premultiplying by the Leontief inverse (the inverse of the Leontief matrix) gives equation (3):

$$\mathbf{q} = (\mathbf{I} - \mathbf{A})^{-1} \mathbf{f}$$

## Annex D: Matrix Algebra for Industrial Analysis

The following notation is used:

**M** is the supply table excluding the columns for imports, margins and taxes less subsidies

**q** is the vector of domestic supply of product IOGs at basic prices

**g** is the row vector of output of industry IOGs

**F** is the matrix of final demand at basic prices

**P** is that part of the matrix of coefficients corresponding to primary inputs

$(\mathbf{I}-\mathbf{A})^{-1}$  is the (product by product) Leontief inverse

Let:

$$\mathbf{D}^* = \mathbf{M}' (\text{diag}(\mathbf{q}))^{-1}$$

$$\mathbf{Q}^* = (\mathbf{I}-\mathbf{A})^{-1} \mathbf{F}$$

$$\mathbf{G}^* = \mathbf{D}^* \mathbf{Q}^*$$

The industrial analysis of primary inputs is then given by:

$$(\mathbf{P} / (\mathbf{1} \mathbf{g})) \mathbf{G}^*$$