Office for National Statistics

Chain-linking methods used within the UK National Accounts

Contact:
Steve Drew, Phil Lewis, and Craig McLaren
philip.a.lewis@ons.gsi.gov.uk

Release date:
10 October 2016

Next release:
To be announced

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1 . Introduction

We produce the UK National Accounts estimates which include the calculation of volume measure series for economic data.

This is a technical reference paper which describes unchaining and chain-linking methodology used in the production of volume measures within the UK. Section 10 describes all terminology used in this paper.

A volume measure is a series of economic data from successive years expressed in real terms by calculating the production volume for each year in the prices of a reference year. The resultant time-series of production figures has the effects of price changes removed. That is, the effects of monetary inflation or deflation have been removed so that the series reflects only production volume.

A current price (CP) estimate records the actual or estimated monetary value for a defined period. The current price estimate is the value expressed in terms of the prices of that period. A time series of CP estimates can be constructed.

If a suitable deflator (or equivalently referred to as a price index) exists, then applying the deflator to a CP series the associated constant price (KP) or sometimes referred to as the volume measure (VM) series can be calculated. The relationship is

value = volume x price

So in this notation

CP = KP x deflator

and then

CP/deflator = KP

Chain volume measure (CVM) estimates are VM estimates obtained by chain-linking. They are the result of joining together two indices that overlap in one period by rescaling one of them to make its value equal to that of the other in the same period, thus combining them into single consistent chain volume measure time series.

Corresponding CP and KP series are unchained to produce the previous year's prices series (PYP) and current year's prices series (CYP). The PYP series may be added or subtracted, weighted together or proportioned out to produce a new PYP series. Similarly the CYP series may be added or subtracted, weighted together or proportioned out. The resultant PYP series with its corresponding CYP series may then be chain-linked to produce a CVM estimate.

Eurostat (2013a, Section 2.2.3) defines the base and reference year as:

"– the base year is the year whose current price values are used to weight the price and volume measures derived at the elementary level of aggregation;

– the reference year is the year which is used for the presentation of a time series of volume data. In a series of index numbers it is the year that takes the value 100."

It is important to note that the reference year bears no fixed relation to the years from which weights have been used and re-referencing is simply the application of a scaling factor to a time series, and does not change the growth rates in the time series.

The expression "chained volume measure" or “CVM” is used in the UK National Accounts publications to describe volume measures derived by chain-linking in either index form (that is, set to be 100 for reference year) or in £million form (that is referenced to the current monetary value in the reference year). This is also sometimes referred to as a “chain-linked volume” (CLV) or “chained volume” (see Eurostat 2013b, Chapter 6). The “CVM” notation is used in this article.

For constant price estimates the base period and the reference period coincide. Importantly for chained volume measures there is only one reference period, but there are many base periods.

The PYP (or CYP) series can be treated in the same way as a fixed base series, as you essentially have a series of annual fixed base series. A fixed base series uses fixed weights from the defined base period for all component series. So in this format PYP may be added or subtracted, weighted together or proportioned out. Chain-linking the PYP series together (using the corresponding CYP series) converts the unchained series which are expressed in the previous year’s prices into a continuous series expressed in the prices of the reference year. The weights of a chained volume measure are updated each year and are based on the previous year’s data. That is, it is not a fixed-weighted index with a single base year. Effectively the chained volume measure has multiple base periods and one reference period.

Additivity over aggregations is lost when these calculations are made for the chained periods. That is component series do not necessarily sum to totals. Eurostat (2013a, Section 2.2.3) also covers this point and states: “To keep all year-to-year growth rates of each variable unchanged when the reference year is changed, one should re-reference each variable separately, be it an elementary index, a sub-total or an overall aggregate such as GDP. The consequence is that, in the chained volume data of a fixed reference year, discrepancies will arise between individual elements and their totals. This is the 'non-additivity' problem”.

In the UK national accounts the reference period used is the last fully balanced year. The choice of reference period in the UK national accounts is discussed in Documentation on Current Methods used for National Accounts, ONS (2008):

“The same reference year is used for all national accounts aggregates. When annual chain-linking was introduced to the UK National Accounts, the decision was made, after consultation with external users, to make the reference year the same as the last year from which weights have been used in the aggregate time series (and adopt this reference year with lower level-time series, regardless of the last year of weights which had been used in their construction).”

Every year we update the UK National Accounts through a process known as annual supply and use balancing. This brings together detailed data on the three approaches to measuring gross domestic product (GDP). It balances supply and demand by product, inputs and outputs by industry. Volume series are updated so their reference and base years are moved forward, usually (but not always) by one year. At the same time, major methodological or classification changes, may be implemented within the accounts. The publication incorporating these changes is known as the Blue Book, and the Quarterly National Accounts publication published a month prior is consistent with Blue Book. The Blue Book also includes changes made to incorporate new data sources, including those used for annual benchmarking purposes.

Weights to aggregate series are created for all years that have been through at least two annual rounds of supply use balancing. After the reference year the weights are not yet available. Therefore for years after the reference year the weights from the reference year are used. Consequently after the reference year the aggregation becomes a fixed base aggregation, as the final year for which weights exist becomes the base year for all years thereafter. This is sometimes referred to as a “fixed base tail”. See section 5 for discussion on the treatment of series tails. For Blue Book 2016, a full set of weights were available for years up and including 2013. That is, 2013 was set as the reference period for Blue Book 2016. (Blue Book 2017 is discussed in section 6).

2 . Unchaining

Unchaining converts a constant price series, for example constant price (KP), volume measure (VM) or chained volume measure (CVM), with a single reference year into an unchained series where each year is expressed in the previous year's prices. In our context the “last base year” is the last year which has been balanced through input-output supply and use tables (see Mahajan, 2006), where all values after that year are expressed in that year's prices. The last base year is used as the reference period and in the reference period the annual current price (CP) value = the annual KP value.

Within our processing system, unchaining is achieved using a standard statistical function, UNCHAIN. The inputs are a KP series and a corresponding CP series. The last base year for these series is stated as a parameter in the function call as well as whether the output should be a previous year’s prices (PYP) or a current year’s prices (CYP) series.

The steps of the process required before unchaining are:

- collect and derive a time series of CP estimates
- calculate a KP equivalent time series, for example, by calculating deflators and then deflating the CP estimates
- for a single KP time series the quarterly values in each year may be added together to generate the annual value. Similarly the monthly values in each year may be added to generate the quarterly values or the annual value
- calculate annual CP values (that is, sum quarterly input (or monthly) to annual)
- calculate annual KP values (that is, sum quarterly input (or monthly) to annual)
- calculate PYPs as described below (that is, unchaining)
- calculate CYPs as described below (that is, unchaining)

Within our processing systems the UNCHAIN function can also be used for intermediate calculations in order to derive the required CVM outputs. The outputs from the UNCHAIN function are CYP and PYP estimates which are consistent so that when chain-linked back again give the input CVM estimate (before benchmarking and smoothing are applied). Note, in this context for national accounts, that chain-linking is not the opposite of unchaining because chain-linking includes additional statistical processing steps for benchmarking and smoothing.

The PYP (and CYP) time series may be used in the same way as a fixed base series. Each year of the PYP series is an annual fixed base series. So that the PYP series may be added or subtracted as well as weighted together or proportioned out as required to the desired series. Similarly this applies to the CYP series.

2.1 Calculating previous year's prices: for years up to and including the last base year

The PYP estimates are calculated from the value of the constant price series for that period divided by the value of the annual constant price series for the year preceding the time period and multiplied by the value of the annual current price series for that year.

To generate the PYP series for “year y” the annual CP and annual KP values for year y-1 are required. Therefore the PYP series starts for the year after the first year of the input CP and KP data. This is sometimes referred to as “losing the first year”.

Note that in the following notation, data for months, quarters or years used are complete full data for that time period. A separate discussion and treatment is needed for the tail of the time series (see Section 2.3 in this paper).

Note that for ease of reading the notation PYP(y) is used to represent the PYP value for year y. This is often written as PYP_y.

2.1.1 Notation for unchaining an annual KP series to a PYP series for a given year

$$PYP(y) = \frac{KP(y)}{KP(y-1)} \times CP(y-1) = KP(y) \times \frac{CP(y-1)}{KP(y-1)}$$

where

- y-1 is the year previous to year y
- PYP(y) is the PYP value for year y
- KP(y) is the KP value year y
- CP(y-1) is the annual CP value for year y-1
- KP(y-1) is the annual KP value for year y-1

2.1.2 Notation for unchaining a quarterly KP series to a PYP series for a given quarter

$$PYP(q,y) = \frac{KP(q,y)}{KP(y-1)} \times CP(y-1) = KP(q,y) \times \frac{CP(y-1)}{KP(y-1)}$$

where

- y-1 is the year previous to year y
- PYP(q, y) is the PYP value for quarter q, year y
- KP(q, y) is the KP value for quarter q, year y
- CP(y-1) is the annual CP value for year y-1
- KP(y-1) is the annual KP value for year y-1

2.13 Notation for unchaining a monthly KP series to a PYP series for a given month

$$PYP(m,y) = \frac{KP(m,y)}{KP(y-1)} \times CP(y-1) = KP(m,y) \times \frac{CP(y-1)}{KP(y-1)}$$

where

- y-1 is the year previous to year y
- PYP(m, y) is the PYP value for month m, year y
- KP(m, y) is the KP value for month m, year y
- CP(y-1) is the annual CP value for year y-1
- KP(y-1) is the annual KP value for year y-1

2.2 Calculating current year's prices: for years up to and including the last base year

The current year’s price series (CYP) is calculated as the value of the constant price series for that period divided by the value of the annual constant price series for the year containing the time period and multiplied by the value of the annual current price series for that year.

2.2.1 Notation for unchaining an annual KP series to a CYP series for a given year

$$CYP(y) = \frac{KP(y)}{KP(y)} \times CP(y) = CP(y)$$

where

- CYP(y) is the CYP value for year y
- KP(y) is the KP value year y
- CP(y) is the annual CP value for year y

2.2.2 Notation for unchaining a quarterly KP series to a CYP series for a given quarter

$$CYP(q,y) = \frac{KP(q,y)}{KP(y)} \times CP(y) = KP(q,y) \times \frac{CP(y)}{KP(y)}$$

where

- CYP(q, y) is the CYP value for quarter q, year y
- KP(q, y) is the KP value for quarter q, year y
- CP(y) is the annual CP value for year y

2.2.3 Notation for unchaining a monthly KP series to a CYP series for a given month

$$CYP(m,y) = \frac{KP(m,y)}{KP(y)} \times CP(y) = KP(m,y) \times \frac{CP(y)}{KP(y)}$$

where

- CYP(m, y) is the CYP value for month m, year y
- KP(m, y) is the KP value for month m, year y
- CP(y) is the annual CP value for year y

2.3 Calculating previous year's prices and current year's prices for periods after the last base year (the tail)

For our outputs up to Blue Book 2016 after the last base year, both the PYP and CYP series are defined to be the value of the constant price series for that period divided by the value of the annual constant price series for the last base year and multiplied by the value of the annual current price series for the last base year. However, by definition of the last base year, the annual constant price series equals the value of the annual current price series for the last base year. Periods after the last base year are often referred to as the tail of the series. Therefore periods in the tail for outputs up to Blue Book 2016, were not truly a “previous year’s prices” series despite being labelled as one.

For Blue Book 2016, the last base year will be set to be 2013. See section 5 for further discussion on the treatment of series tails.

2.3.1 Notation for CYP series and PYP Series from annual KP for periods after the last base year for a given year

$$PYP(y) = CYP(y) = KP(y)$$

where

- PYP(y) is the PYP value for year y
- CYP(y) is the CYP value for year y
- KP(y) is the KP value year y

2.3.2 Notation for CYP series and PYP Series from quarterly KP for periods after the last base year for a given quarter

$$CYP(q, y) = PYP(q, y) = KP(q, y)$$

where

- PYP(q, y) is the PYP value for quarter q, year y
- CYP(q, y) is the CYP value for quarter q, year y
- KP(q, y) is the KP value for quarter q, year y

2.3.3 Notation for CYP series and PYP Series from monthly KP for periods after the last base year for a given month

$$CYP(m, y) = PYP(m, y) = KP(m, y)$$

where:

- PYP(m, y) is the PYP value for month m, year y
- CYP(m, y) is the CYP value for month m, year y
- KP(m, y) is the KP value for month m, year y

Algebraically, for example, for a quarterly KP input series:

If LBY = the last base year, then for a chosen year greater than the last base year, for example y > LBY:

$$PYP(q,y) = CYP(q,y) = \frac{KP(q,y)}{KP(LBY)} \times CP(LBY)$$
$$= KP(q,y) \times \frac{CP(LBY)}{KP(LBY)}$$

where

- LBY is the last base year
- PYP(q, y) is the PYP value for quarter q, year y
- CYP(q, y) is the CYP value for quarter q, year y
- KP(q, y) is the KP value for quarter q, year y
- CP(B) is the annual CP value for the last base year
- KP(B) is the annual KP value for the last base year

By definition of the last base year

$$CP(LBY) = KP(LBY)$$

And KP(LBY) is non-zero by definition, so

$$\frac{CP(LBY)}{KP(LBY)} = 1$$

Therefore,

$$PYP(q,y) = CYP(q,y) = KP(q,y) \times 1 = KP(q,y)$$

A similar approach is used for both the annual and the monthly KP input series cases.

3 . Deriving weights for aggregation

Chain-linking calculations are typically performed at the lowest level possible. Aggregates then need to be formed from the lower-level calculations, which are achieved through the use of weights.

The weights of a chain volume measure are updated each year and are based on the fully available previous year’s data. That is, the chain volume measure is not a fixed-weighted index with a single base year.

Chain Linked Weights, ONS (2014a) notes that: “Weights for 112 industry groups are calculated based on the contribution of each industry to the overall economy. These are derived from gross value added (GVA) totals produced by ONS in the Supply and Use Tables. GVA for an industry group is divided by total GVA across the whole economy and then multiplied by 1000, to give a parts per thousand weight for that industry group. Weights are created for all years that have been through at least 2 annual rounds of supply use balancing. The last year we have weights for is also used as the reference year for the index (this is the convention for the UK but does not have to be the case) … previous years weights are revised for the open period.”

See section 5 for discussion on the treatment, including the weighting, of series tails.

4 . Chain-linking

For our outputs up to Blue Book 2016, the last base year is used as the reference year. From Blue Book 2017, the chain-linking calculations within our systems will still process using the last base year as the reference year but will do additional calculations to re-reference to a distinct separate reference year.

Within the UK National Accounts, we use Laspeyres chain-linking (Eurostat 2013b, Chapter 6). The defining feature of Laspeyres is that in calculating growth from one period to another, the prices of the earlier period are applied to both periods. The choice of an annually-chained Laspeyres index as the formula for computing annual growth rates is straightforward and unambiguous. However, the specific linking methodology has to take into account a number of factors including consistency with other series, monthly path and annual growth rates.

The methods used in our UK National Accounts are explained in later sections. However, the technical decisions made for introducing annual chain-linking into our UK National Accounts appear in box 5 of Tuke and Reed (2001), copied below:

- Annual chain-linking will be carried out using Laspeyres indices and series will be referenced to the last weights year to preserve additivity in the most recent data
- Annual chain-linked quarterly data will be linked using an overlap on quarter four. Any resultant drift away from the annual growth path will be removed by benchmarking to annual data. The decision was made to use a quarterly overlap method rather than alternatives suggested by Eurostat because it best preserves the quarterly growth path
- Annual chain-linked monthly data will also be linked on quarter 4 and benchmarked to annual data. This will minimise the impact of December to January effects and provides estimates which follow the quarterly growth path
- The level at which annual chain-linking will be implemented for GDP(O) will meet Eurostat requirements and will be implemented at an even lower level where constant price data is consistently available and modelling has shown that this chain-linking may significantly alter growth estimates
- The monthly Index of Production will also be annual chainlinked to give consistency between monthly, quarterly and annual published estimates

It should be noted that our Prices area derive price information using a different but appropriate methodology. For further details Consumer Price Indices Technical Manual, ONS (2014b) and Ralph et. al. (2015).

4.1 Notation for annual series chain-linking for a given year

The annual chained volume measure (CVM) is defined

$$CVM(y) = \begin{cases} CYP(y) \times \prod_{y<t\leq LBY} \frac{CYP(t)}{PYP(t)} & \text{if } y < LBY, \\ CYP(y) & \text{if } y = LBY, \\ CYP(y)\,(= PYP(y)) & \text{if } y > LBY. \end{cases}$$

For low level series in may be possible that for a particular year CYP(y) = 0 and / or PYP(y) = 0. If for a particular year CYP(y) = 0 and / or PYP(y) = 0, then set

$\frac{CYP(y)}{PYP(y)} = 1$

4.2 Notation for quarterly series chain-linking

A consistent approach to chain-linking has to be taken for the quarterly national accounts and short-term volume indicators (such as the Index of Production, Output in the Construction Industry, the Index of Services, and the Retail Sales Index), in order to ensure consistency between the different series. The quarterly path is best calculated using the quarterly overlap technique (with quarter 4 (Oct to Dec) being the link period), as using an annual overlap can introduce a jump between quarter 4 of one year and quarter 1 (Jan to Mar) of the next year (for example, in Chapter 9 of Bloem et. al (2001) – in particular Example 9.4.a. on page 159.).

Applying the quarterly overlap to a monthly series ensures consistency with the quarterly series, while maintaining a smooth monthly path. However, on its own, a quarterly overlap can lead to divergence between the monthly (or quarterly) series and the annual series. Thus, the monthly (or quarterly) series is benchmarked to the annual series in order to ensure consistency with the annual series and remove any divergence. This is done automatically with the chain-linking function within our production systems. Consequently, quarterly overlap with benchmarking, combines the principal advantages of both quarterly overlap (best intra-year path) and annual overlap (consistency with annual data), and meets the requirements of key users of the data.

There is considerable discussion in the literature around aspects of the quarterly overlap method. For example, Eurostat (2013b, Section 6.51) states:

“A key property of the one-quarter overlap method is that it preserves the quarter-on-quarter growth rate between the fourth quarter of year y-1 and the first quarter of year y – unlike the annual overlap method. The ‘damage’ done to that growth rate by the annual overlap method is determined by the difference between the annual and quarter overlap link factors. Conversely, this difference also means that the sum of the linked quarterly values in year y-1 differ from the annual overlap-linked data by the ratio of the two link factors. Temporal consistency can be achieved by benchmarking the quarterly chain-linked volume estimates derived using the one-quarter overlap method to their annual counterparts. By using an optimal benchmarking procedure (see Chapter 5), the difference between the two link factors for each year is spread over many quarters such that the amendments to the quarter-on-quarter movements are minimized.”

Section 9.41, page 158, Bloem et. al (2001), states:

“To conclude, there are no established standards with respect to techniques for annually chain-linking of QNA data, but chain-linking using the one-quarter overlap technique, combined with benchmarking to remove any resulting discrepancies between the quarterly and annual data, gives the best result. In many circumstances, however, the annual overlap technique may give similar results. The over-the-year technique should be avoided.”

As per standard chain-linking approaches, both the current year’s price (CYP) and price year’s price (PYP) quarterly series are additive. The annual value may be obtained from summing the individual quarters.

Hence,

$$CYP(y) = \sum_{Q1 \le q \le Q4} CYP(q,y) \text{ and } PYP(y) = \sum_{Q1 \le q \le Q4} PYP(q,y)$$

The Unconstrained CVM series is an intermediate series that is produced before being constrained by benchmarking to the annual CVM series to produce the final CVM.

$$\text{Unconstrained } CVM(q,y) =$$

$$\begin{cases} PYP(q,y) \times \prod_{y \le t \le LBY} ratio(Q4,t) \text{ if } (y \le LBY) \text{ and q} \in \{\text{Q1, Q2, Q3}\}, \\ CYP(q,y) \times \prod_{y < t \le LBY} ratio(Q4,t) \text{ if } (y < LBY) \text{ and } q = Q4, \\ CYP(q,y) \text{ if } (y = LBY, Q4), \\ CYP(q,y) \; (= PYP(q,y)) \text{ if } (y > LBY). \end{cases}$$

Where the ratio

$$Ratio(Q4,y) = \begin{cases} \frac{CYP(Q4,y)}{PYP(Q4,y)} \\ \text{if } CYP(Q4,y) \neq 0 \text{ or } PYP(Q4,y) \neq 0 \\ \text{or } CYP(y) \neq 0 \text{ or } PYP(y) \neq 0, \\ \frac{CYP(y)}{PYP(y)} \\ \text{if } CYP(y) \neq 0 \text{ and } PYP(y) \neq 0 \text{ and} \\ (CYP(Q4,y) = 0 \text{ or } PYP(Q4,y) = 0), \\ 1 \\ \text{if } CYP(y) = 0 \text{ and/or } PYP(y) = 0. \end{cases}$$

and where LBY = last base year.

In the standard situation where the annual series and quarter 4 (Oct to Dec) values for both the PYP and CYP are never zero, for example in the case of high level aggregates, then the algebra may be simplified to

$$\text{Unconstrained } CVM(q,y) =$$

$$\begin{cases} PYP(q,y) \times \prod_{y \leq t \leq LBY} \frac{CYP(Q4,t)}{PYP(Q4,t)} \text{ if } (y \leq LBY) \text{ and } q \in \{Q1,Q2,Q3\}, \\ \text{CYP(q,y)} \times \prod_{\text{y<t}\leq \text{LBY}} \frac{CYP(Q4,t)}{PYP(Q4,t)} \text{ if } (y < LBY) \text{ and } q = Q4, \\ CYP(q,y) \text{ if } (y = LBY, q = Q4), \\ CYP(q,y) \, (= PYP(q,y)) \text{ if } (y > LBY). \end{cases}$$

An annual version of the input series are then calculated, then these annual series are then chain linked using the method in section 4.1.

The quarterly unconstrained CVM is then benchmarked to the annual chain linked series to ensure consistency.

An important user requirement is that the quarterly system of accounts should provide high quality measures of growth, with no discontinuities, and it should allow for growth to be estimated over varying period lengths. The Cholette-Dagum method (Cholette and Dagum, 1994), a special case of Denton, is now the default method used for benchmarking within our National Accounts. The constrained chained volume measure is the benchmarked series for time periods up to and including the last base year. The constrained chained volume measure is the unconstrained chained volume measure for time periods after the last base year.

4.3 Notation for monthly series chain-linking

The UK short-term volume indicators (for example the Index of Production, Output in the Construction Industry, the Index of Services, and the Retail Sales Index) are fully integrated with the quarterly and annual national accounts; that is, monthly estimates are constrained to sum to annual estimates, both in terms of current prices and CVMs. It is therefore an important requirement for the UK that the following basic accounting conventions are preserved within a chain-linked system.

As per standard practice, both the CYP and PYP series are additive. A quarterly value may be obtained from summing the associated monthly values. For example,

$$CYP(Q4,y) = \sum_{m=\text{Oct,Nov,Dec}} CYP(m,y)$$

and

$$PYP(Q4,y) = \sum_{m=\text{Oct,Nov,Dec}} PYP(m,y)$$

In this situation the unconstrained CVM is defined as:

$$\text{Unconstrained } CVM(m,y) =$$

$$\begin{cases} PYP(m,y) \times \prod_{y \leq t \leq LBY} ratio(Q4,t) \text{ if } (y \leq LBY) \text{ and } m \in \{\text{Jan to Sep}\}, \\ CYP(m,y) \times \prod_{y \leq t \leq LBY} ratio(Q4,t) \text{ if } (y < LBY) \text{ and } m \in \{\text{Oct to Dec}\}, \\ CYP(m,y) \text{ if } (y = LBY, m \in \{\text{Oct to Dec}\}, \\ CYP(m,y) \, (= PYP(m,y)) \text{ if } (y > LBY). \end{cases}$$

Where the ratio

$$Ratio(Q4,y) = \begin{cases} \frac{\frac{CYP(Q4,y)}{PYP(Q4,y)}}{\text{if } CYP(Q4,y) \neq 0 \text{ or } PYP(Q4,y) \neq 0 \\ \text{or } CYP(y) \neq 0 \text{ or } PYP(y) \neq 0,} \\ \frac{\frac{CYP(y)}{PYP(y)}}{\text{if } CYP(y) \neq 0 \text{ and } PYP(y) \neq 0 \text{ and} \\ (CYP(Q4,y) = 0 \text{ or } PYP(Q4,y) = 0),} \\ 1 \\ \text{if } CYP(y) = 0 \text{ and/or } PYP(y) = 0. \end{cases}$$

and where LBY = last base year.

An annual version of the input series can be calculated, where these annual series are then chain linked using the method in 4.1. The quarterly unconstrained CVM is then benchmark to the annual chain linked series. Similar to the quarterly benchmarking described in 4.2.

5 . Tail of series up to Blue Book 2016

Up to and including the last base year we have full years of data and weights, the calculations for the estimates follow the forms as described in the earlier sections. After the last base year a fixed base volume calculation is used for the chained volume measure (CVM) series – this is referred to as the “tail” or the “Fixed Base Tail”. In effect two methods are joined together in the last base year.

Our treatment of the tail is covered in Documentation on Current Methods used for National Accounts, ONS (2008, Section 2.8.2) which notes:

“The same reference year is used for all National Accounts aggregates. When annual chain-linking was introduced to the UK National Accounts, the decision was made, after consultation with external users, to make the reference year the same as the last year from which weights have been used in the aggregate time series (and adopt this reference year with lower level time series, regardless of the last year of weights which had been used in their construction). This allows for additivity in the latest time periods, i.e. component levels add to aggregate levels for £m series, and for index series the aggregates can be calculated by weighting together the separate components. This additivity is found in the reference year onwards for annual levels and in the reference year plus one onwards for quarterly and monthly levels. In contrast with the use of CVMs with output or consumption valued at the prices in the reference year, when volume changes are measured relative to a fixed base year, the volume series re-valued at the prices in the base year are termed constant price (KP) estimates.”

The calculation of the tail of the series is not covered explicitly in the international manuals. However, Eurostat (2013b, Sections 6.85 and 6.86) states:

“The requirements to derive estimates for a quarter in the prices of the previous year are volume indices from the base year to the quarter and price (or current price) data for the base year – see formula 6.14. These requirements are always met for the final expenditure components of GDP(E), but the price data may not be available for the gross value added by industry components of GDP(P). This is likely to be the case if quarterly current price estimates of gross value added are not derived and the quarterly volume estimates of gross value added are derived, in the main, by extrapolation using volume indicators of output. In such cases, the timing of the introduction of a new base year may need to coincide with the introduction of new annual estimates, including first estimates of gross value added for the new base year.

“The timing and consequences of this approach is best explained using an example. Suppose in country A new annual estimates and a new base year are introduced with the release of data for the second quarter each year. Further suppose the current year is year y and the new annual estimates are in respect of year y-2, then country A may choose to make the year y-2 the new base year. This means that (prior to linking) the volume estimates for the four quarters in year y-1 and the first two quarters of year y are now derived in the average prices of year y-2. By contrast, in the preceding release (that is that for the first quarter of year y) the volume estimates for the four quarters of year y-1 and the first quarter of year y were derived in the average prices of year y-3. The change of base year, from year y-3 to year y-2, is likely to change the growth rates of the five quarters concerned. For the majority of national accounts statistics, price and volume relativities do not change very much between one year and the next, and so in most cases the revisions to growth rates are small.”

Within our context, for Blue Books up to Blue Book 2016, the last base year is the same as the reference year. For example, for Blue Book 2016, the last base year is 2013. In which case the

CVM series = CYP series = PYP series

for periods after 2013. This holds for monthly, quarterly and annual series. As noted previously, the previous year' s prices (PYP) series is defined the same as current year's prices (CYP) series after the last base year and so is thus not truly "previous year's prices" despite the name.

The tail is a constant price series as it is based on fixed-price weights which are the same as those used for the last base year, 2013.

For Blue Book 2017 onwards, section 6 describes the calculations required in relation to the tail estimates.

6 . Re-referencing and the tail calculation from Blue Book 2017

6.1 Background on tail calculation

Drew et. al (2016) describes this issue in more detail and provides an explanation of the change in approach to UK outputs.

ESA 2010 (Eurostat, 2013c) states that chained volume measures (CVMs) should be formed the prices of the previous year. For the UK, the chain-linking function has two methods available, the correct annual chain-linking method up until the last base year and a fixed base method for the tail, which was defined when annual chain-linking was first introduced in 2003.

The issue of previous year's prices was not included in ESA 1995 (Eurostat, 1999) and was covered instead by the Price and Volume Handbook in 2001 (Eurostat, 2001). This handbook mandated previous years for annual data only and became a legislative requirement from 2006 for the UK. The Quarterly National Accounts Handbook is the only Eurostat handbook to mention the issue of the tail, and sets out the option of having a tail. ESA 2010 sets out in 10.20:

"Therefore, the calculation of the volume is made only for two successive years, that is the volume is calculated at the prices of the previous year."

There is no discussion of the practical issues surrounding this implementation and has been interpreted as applying to all years. Eurostat (2013a) makes no changes in this area to the 2001 version.

6.2 Calculations for the Reference Year from Blue Book 2017 onwards

From Blue Book 2017 onwards the reference year and last base year will be able to be distinct and different.

In practice, the reference year is a presentational reference point where the annual CP = annual CVM.

From Blue Book 2017 the reference year is the first year balanced in the supply and use tables (SUT) framework. For example, for Blue Book 2017 the Reference Year will be 2015 which is the initial balanced year in SUT (t-2) and 2014 is the re-balanced year (t-3).

Note that many other countries keep their reference year fixed for many years. Some countries update in line with large benchmarking exercises every five years.

6.3 Changes to UK production systems

UK production systems will continue to process constrained CVMs as previously used (and described in Sections 2 to 6 in this note) and will re-reference the constrained CVMs to desired reference year.

6.3.1 Calculations for re-referencing annual series

The re-referencing is achieved by the calculation:

$$CVMRE(y) = \frac{CP(ref)}{CVM(ref)} \times CVM(y)$$

where

- ref is the reference year
- CVMRE(y) is the annual referenced CVM value for year y
- CVM(ref) is the annual CVM value for the reference year
- CP(ref) is the annual CP value for the reference year
- CVM(y) is the annual CVM value for year y

6.3.2 Calculations for re-referencing quarterly series

The re-referencing is achieved by the calculation:

$$CVMRE(q, y) = \frac{CP(ref)}{CVM(ref)} \times CVM(q, y)$$

where

- ref is the reference year
- CVMRE(q, y) is the referenced CVM value for quarter q, year y
- CVM(ref) is the annual CVM value for the reference year
- CP(ref) is the annual CP value for the reference year
- CVM(q, y) is the CVM value for quarter q, year y

6.3.3 Calculations for re-referencing monthly series

$$CVMRE(m, y) = \frac{CP(ref)}{CVM(ref)} \times CVM(m, y)$$

where

- ref is the reference year
- CVMRE(m, y) is the referenced CVM value for month m, year y
- CVM(ref) is the annual CVM value for the reference year
- CP(ref) is the annual CP value for the reference year
- CVM(m, y) is the CVM value for month m, year y

6.4 Approach to generate previous year price's that are consistent with re-referenced constrained CVM series and with the desired tail properties

Due to the way our production systems are configured the most efficient way to generate the required version of the previous year price's series is to perform additional calculations to meet the required properties.

The notation CVMRE gives the constrained CVM values that have been referenced to a reference year that is distinct from the last base year.

From Blue Book 2017 onwards the CVMRE series are the published chain-linked estimates. The corresponding PYP series published, which relate to the CVMRE series, are described below.

6.4.1 Calculations for previous year's price annual series aligned to CVMRE

The previous year's price based on this reference year is calculated as:

$$PYP*(y) = \frac{CVMRE(y)}{CVMRE(y-)} \times CP(y-1)$$
$$= CVMRE(y) \times \frac{CP(y-1)}{CVMRE(y-1)}$$

where

- y-1 is the year previous to year y
- PYP*(y) is the annual PYP value for year y consistent with the referenced values
- CVMRE(y) is the referenced CVM value for year y
- CP(y-1) is the annual CP value for year y-1
- CVMRE (y-1) is the referenced CVM value for year y-1

6.4.2 Calculations for previous year's price quarterly series aligned to CVMRE

The previous year's price based on this reference year is calculated as:

$$PYP*(q,y) = \frac{CVMRE(q,y)}{CVMRE(y-1)} \times CP(y-1)$$
$$= CVMRE(q,y) \times \frac{CP(y-1)}{CVMRE(y-1)}$$

where

- y-1 is the year previous to year y
- PYP*(q, y) is the PYP value for quarter q, year y consistent with the referenced values
- CVMRE(q, y) is the referenced CVM value for quarter q, year y
- CP(y-1) is the annual CP value for year y-1
- CVMRE (y-1) is the referenced CVM value for year y-1

6.4.3 Calculations for previous year's price monthly series aligned to CVMRE

The previous year's price based on this reference year is calculated as:

$$PYP*(m,y) = \frac{CVMRE(m,y)}{CVMRE(y-1)} \times CP(y-1)$$
$$= CVMRE(m,y) \times \frac{CP(y-1)}{CVMRE(y-1)}$$

where

- y-1 is the year previous to year y
- PYP*(q, m) is the PYP value for month m, year y consistent with the referenced values
- CVMRE(q, m) is the referenced CVM value for month m, year y
- CP(y-1) is the annual CP value for year y-1
- CVMRE (y-1) is the referenced CVM value for year y-1

6.5 Property of the previous year price's for the year after the reference year

If y is the year after the reference year, that is y = ref +1, then

$$PYP*(q,ref+1) = CVMRE(q,ref+1) \times \frac{CP(ref)}{CVMRE(ref)}$$
$$= CVMRE(q,ref+1) \times 1 = CVMRE(q,ref+1)$$

Where

- ref is the reference year
- PYP*(q, y) is the PYP value for quarter q, year y consistent with the referenced values
- CVMRE(q, y) is the referenced CVM value for quarter q, year y
- CVM(ref) is the annual CVM value for the reference year
- CP(ref) is the annual CP value for the reference year
- CVM(q, y) is the CVM value for quarter q, year y

That is, in the year after the reference year the previous year's price value is the same as re-referenced constrained volume measure value.

7 . National Accounts Identities and consistency checks

Published National Accounts estimates should always adhere to standard identities. The following sections outline identities relevant to chained volume measure (CVM) and current price (CP) estimates.

7.1 Chain-linking one CYP series with a corresponding PYP series where the last base year is the reference year. Where in the tail the PYP series = CVM series = CYP series. That is, the method used up to Blue Book 2016

- Conceptually, the CP series and current year’s prices (CYP) series are not the same
- Annual CP series = annual CVM in the last base year
- Annual CVM = annual CYP in the last base year and subsequent periods
- In the last base year the monthly CVM components sum to the annual CVM value
- In the last base year the quarterly CVM components sum to the annual CVM value
- In the last base year or for periods before the last base year: monthly CVM components do not necessarily add to the quarterly CVM value
- After the last base year CVM series are additive, that is, monthly may be summed to quarterly or annual, quarterly may be summed to annual
- CVM series = previous year’s prices (PYP) series = CYP series post the last base year. This holds for monthly, quarterly and annual series
- CYP and PYP individual time series are additive over time, that is, monthly may be summed to quarterly or annual, quarterly may be summed to annual
- PYP series actually use prices of the last base year for all years after the last base year (so not "previous year’s prices" in the tail despite the name). That is, the tail is actually the CYP series
- Annual CP = annual CYP up to and including the last base year

7.2 Aggregation of series where the last base year is the reference year. Where in the tail the PYP series = CVM series = CYP series. That is, the method used up to Blue Book 2016

- CVMs are additive post the last base year, that is, after the last base year lower level component CVM time series, can be weighted and summed to create aggregate a higher level CVM time series
- In the last base year or for periods before the last base year: lower level component CVM series are not additive, that is, lower level component CVM time series, cannot be weighted and summed to create aggregate higher level CVM time series
- PYP series are always additive, that is, lower level component PYP time series, can be weighted and summed to create aggregate higher level PYP time series
- CYP series are always additive, that is, lower level component CYP time series, can be weighted and summed to create aggregate higher level CYP time series
- Annual SA totals are forced to equal annual NSA totals, our convention, in line with international best practice (for example the QNA manual) is that annual totals for seasonally adjusted series are forced by benchmarking to equal annual totals of the corresponding non-seasonally adjusted series

7.3 Chain-linking one CYP series with a corresponding PYP series where the last base year is distinct from the reference year. That is, the method used from Blue Book 2017

When the last base year is distinct from the reference year, CVMRE are constrained CVM values that have been referenced to a reference year that is distinct from and later than the last base year.

- Conceptually, the CP series and CYP series are not the same
- Annual CP series = annual CVMRE in the reference year
- Annual CP = annual CYP in the reference year
- In the last base year and reference year CVMRE series are additive, that is, monthly may be summed to quarterly CVMRE value or annual CVMRE value, quarterly CVMRE may be summed to annual CVMRE value
- For periods that are not in the last base year or reference year: monthly CVMRE components are not additive, so will not necessarily sum to the quarterly CVMRE or the annual CVMRE value
- For periods that are not in the last base year or reference year: quarterly CVMRE components are not additive, so will not necessarily sum to the annual CVMRE value
- CVM series = PYP series in the year after the reference year. (PYP as in 6.5.)
- CYP and PYP individual time series are additive over time. That is, monthly may be summed to quarterly or annual, quarterly may be summed to annual

7.4 Aggregation of series where the last base year is distinct from the reference year. That is, the method used from Blue Book 2017

When the last base year is distinct from the reference year, CVMRE are constrained CVM values that have been referenced to a reference year that is distinct from the last base year.

- CVMRE are additive for the reference year and the last base year, that is, in the reference year or the last base year level lower component CVMRE time series, can be weighted and summed to create aggregate a higher level CVMRE time series

- For periods other than the reference year or the last base year: lower-level component CVMRE series are not necessarily additive, that is, lower-level component CVMRE time series cannot necessarily be weighted and summed to create aggregate higher level CVMRE time series

- PYP series are always additive, that is lower level component PYP time series, can be weighted and summed to create aggregate higher level PYP time series

- CYP series are always additive, that is lower level component CYP time series, can be weighted and summed to create aggregate higher level CYP time series

- Annual SA = annual NSA totals, that is, annual totals for seasonally adjusted series are forced to equal annual totals of the corresponding non-seasonally adjusted series.

8 . Summary

The same chain-linking methodology is used consistently throughout the UK national accounts. To chain-link quarterly (or monthly) series the "quarterly overlap" technique is used with benchmarking to the annual chain-linked series. It is internationally recognised as the method which gives the best results, with a smooth quarter 4 to quarter 1 transition and annual growth rates which are consistent with the annual series.

This is also recommended as part of the 2008 edition of the United Nations System of National Accounts (UN, 2008) states in paragraph 15.50: “…, chaining using the one-quarter overlap technique with benchmarking to remove any resulting discrepancies between the quarterly and annual data gives the best result.”

9 . Acknowledgements

The authors would like to thank all our colleagues, including Rob Bucknall, Pete Lee, and David Matthews, for their feedback and comments in preparing this article. Of course, any errors and omissions are our responsibility.

10 . Appendix ONS National Accounts notation and terminology

Term	Description

Base Year /Period	The year /period in which volume series are expressed using the value of the average prices.
CP	Current price. Estimates valued in the prices of the period when the activity occurred. Also referred to as “Nominal Prices”.
CVM	Chained volume measure. The result of joining together two indices that overlap in one period by rescaling one of them to make its value equal to that of the other in the same period, thus combining them into single time series. Also referred to as “Chain-Linked Values”.
CVMRE	Chained volume measure that has been re-referenced to a reference year distinct from the last base year. A volume measure obtained by chain linking.
CYP	Current year's prices. The average price in the year in which the activity took place.
Deflator	A price index. By applying the deflator to a current price series the associated volume measure series may be calculated.
Fixed Base Index	An index which uses fixed weights from the defined base period for all component index numbers.
KP	Constant Price Series (KP). Term referring to expressing values in terms of a base period. A volume also known as constant prices or real growth. Calculated directly or indirectly by dividing current price value by the deflator (price index). In effect you are holding the prices constant in the base period or removing the effect of price change.
Last base year	This is the last year in a Chained Volume Measure Series which is used as a base year. In the last base year annual CP value = annual KP value.
NSA	Non-seasonally adjusted.
PYP	Previous year's prices. The average price in the year preceding the period in which the activity took place. The series derived from multiplying the volume series in the current period, by the average price of the previous year. This process is also known as unchaining. Previous Years Prices are additive and are the method by which volume series can be aggregated together.
PYP*	Previous year's prices that are consistent with the re-referenced CVM and the tail are genuine previous year's prices.
Reference period	The reference period is the period for which an index series is set equal to 100 or the period for which a volume index series may be set equal to the current price value in order to express the index series in terms of currency units. In the reference year, the implied deflator is equal to 1, and series are additive.

SA	Seasonally Adjusted. Seasonal adjustment aids interpretation by removing effects associated with the time of the year or the arrangement of the calendar, which could obscure movements of interest.
Volume Index	At the elementary level (that is, detailed disaggregated level) a volume index is most commonly presented as a weighted average of the proportionate changes in the quantities of a specified set of goods or services between two periods of time. In a volume measure, the estimates for all periods are in the same price.
VM	Volume measure.

11 . References and further reading

Bloem, A.M, Dippelsman, R.J., and Maehle N.O. (2001) Quarterly National Accounts Manual—Concepts, Data Sources, and Compilation, International Monetary Fund 2001.

Butler, R. (2012) Chain-linking of UK short term volume series, Office for National Statistics, 2012 Internal Paper.

Cholette, P.A. and Dagum, E.B. (1994). Benchmarking time series with autocorrelated survey errors, International Statistical Review 62, 3: 365-377.

Daniel, E. (2014) An introduction to reconciled estimates of GDP, ONS, March 2014.

Drew, S., Hughes, M., and Denley H., (2016) Changes to Chain Volume Measure calculations for UK Publications and International Transmissions from Blue Book 2017, Office for National Statistics, October 2016.

Eurostat, (2013a), Handbook on Prices and Volume Measures in National Accounts.

Eurostat, (2013b), Handbook on Quarterly National Accounts.

Eurostat, (2013c), European system of accounts - ESA 2010.

Eurostat, (1999), European system of accounts ESA 1995.

Eurostat, (2001), Handbook on price and volume measures in national accounts.

Lee, P. (2011). United Kingdom National Accounts – a short guide, Office for National Statistics, August 2011.

Office for National Statistics (2008), Documentation on Current Methods used for National Accounts, April 2008, UK Centre for the Measurement of Government Activity.

Office for National Statistics (2014a), Chain Linked Weights, January 2014.

Office for National Statistics, (2014b), Consumer Price Indices Technical Manual.

Office for National Statistics (2015a), Economic Review, Office of the Chief Economic Adviser. Office for National Statistics, 05 August 2015.

Office for National Statistics (2015b), Economic Review, Illustrative Example of a CVM Calculation in National Accounts.

Mahajan, S. (2006), Development, compilation and use of Input-Output Supply and Use tables, Office for National Statistics, Economic Trends 634, September 2006.

Ralph, J. O'Neill R., and Winton J. (2015), A Practical Introduction to Index Numbers, ISBN: 978-1-118-97781-1

Robjohns J., Methodological Note: Annual chain-linking, ONS, Economic Trends, No 630, 2006.

Scheiblecker, M., 2010, Chain-linking in Austrian quarterly national accounts and the business cycle, OECD Journal: Journal of Business Cycle Measurement and Analysis, No.: 5, Volume: 2010, Issue: 1 Pages 1–12.

Tuke A. and Reed, G. (2001), The effects of annual chain-linking on the output measure, Office for National Statistics, Economic Trends.

United Nations, 2008, System of National Accounts – SNA 2008.