Methodology of the Monthly Index of Services

Annex A: Mathematical Formulation of the Index of Services

What are index numbers?

An index number is a convenient form of expressing a series in a way that makes it easier to see changes in that series. The items in the series are expressed relatively, with one item in that series chosen to be the 'base' and other items being measured relative to that base. Index numbers have the advantage that they allow different types of data to be combined on a consistent basis, e.g. deflated current price data with volume data.

The Index of Services

The aim of the Index of Services is to provide a reliable indicator of short-term changes in the gross value added of the service sector of the economy. Index numbers are the best medium for communicating this message.

Direct measurement of gross value added is only possible if both outputs and inputs can be measured. This is generally impractical for a monthly indicator, so the assumption is made that, in the short term, changes in output alone represent changes in gross value added. The IoS is therefore designed to measure the volume of output.

Current price figures measure value. The prices are current prices because, for each year, they use that year's prices. Turnover data, for example, are current price figures.

value = price x quantity

Constant prices are used to show how the quantity or volume of goods changes. As a result they are often called **volume measures**. The IoS is a constant price volume index.

quantity = value / price

The volume of output can therefore be measured by dividing current price turnover data by a suitable price indicator. This process is known as deflation.

In parts of the IoS, direct volume measures are used instead of deflated turnover, e.g. volume of shipping freight. In other parts, other indicators are used to proxy output, e.g. input measures, such as number of employees (used as a proxy for some public sector services). All of these alternative measures can be assumed to conform to the same mathematical formula as that for deflated turnover, with the price component held at a constant base value of 100.

Mathematical derivation of the Index of Services

One of the common ways of showing volume figures is to express them at the constant prices of some fixed base year. The formula for this can be written in mathematical notation as:

$$KP_t = P_o * Q_t$$

where KP_t is the value in period t expressed in the constant prices P_o of the base period 0 and Q_t is the quantity in period t.

An advantage of this approach is that the resulting values can be aggregated in the same way as current prices. Also, they are easily understood as what current purchases would have cost in the base year.

In order to calculate constant prices, there are three methods - revaluation, deflation and volume extrapolation - each appropriate to different circumstances.

Revaluation

The most straightforward way to obtain the constant price value for a specific product or service is to multiply the physical quantity produced or performed in the current period by its unit price in the base period. This method has the advantage of not requiring values for any period other than the base year. However, it is not used in the IoS because it requires the collection of data on physical quantities for a vast range of individual services. This would be difficult, expensive, time-consuming and would impose an unacceptable burden on businesses that supply data.

Deflation

Collecting prices for every different type of service would be impractical, so in practice the ONS collects information on a reduced number of indicative prices that are representative of the general movement in prices. These price movements can then be used to construct a price index, $(P_t / P_o) * 100$. It has already been shown that:

$$\text{value} = \text{price} \times \text{quantity}$$

i.e.

$$CP_t = P_t * Q_t$$

So, by dividing through by the price index and then multiplying by 100, a constant price series is obtained,

i.e.

$$\frac{P_t * Q_t}{(P_t/P_o)*100} * 100$$

$$= \frac{P_o Q_t}{100} * 100$$

$$= P_o * Q_t = KP_t, \text{ the constant price value.}$$

This method is widely used in the IoS, for services where current price turnover and suitable price deflators (e.g. Retail Price Indices and Corporate Services Price Indices) are available.

Volume extrapolation

The final method is to update the base year's value using an appropriate volume index. This method has been judged by Eurostat (the statistical office of the European Community) to be generally inferior to deflated turnover as a measure of gross value added, but it is commonly used when deflated turnover data are not available. It only requires a base year value and a series that is indicative of the change in volume.

In the IoS it is used in preference to other proxy indicators (e.g. input measures such as employment numbers) to derive an output measure for services in which values are not readily available for current periods.

So, it is possible to generate constant price series with very little data. However, because the IoS uses the growth in output to approximate for that in gross value added, output volume figures are only shown as indices, relating the value at time t to the average value in the base year, which is set to equal 100,

i.e. $$\frac{(Q_t P_o)}{(Q_o P_o)} * 100$$

This gives a volume index for a single product or service. However, the IoS is comprised of many different services. It is therefore necessary to construct an index that recognises the relative importance of different services, and this is done using weights. Determining an appropriate weighting pattern is a key problem in the construction of index numbers.

The Index of Services is a Laspeyres index. The weights for the IoS are calculated from Current Price Input-Output (CPIO) supply and use tables. For more information about these tables, see the section on **National Accounts**.

Applying base year weights to the formula provides a simple idealised representation of the IoS:

$$S_t = \frac{\sum_{i \in S} \left(w_i \left(\frac{Q_{it} P_{i0}}{Q_{i0} P_{i0}} \right) * 100 \right)}{\sum_{i \in S} w_i}$$

where S_t is the index value at time t; and
w_i is the weight of component i in the index.

This formula assumes complete knowledge of prices, quantities and weights. However, in practise all these elements have to be estimated. This leads to errors in the calculated index, compared to the 'true' index based on complete knowledge. The process of compiling the IoS includes the facility to make quality adjustments at various stages, with the aim of minimising these errors. Because the adjustments are applied at various different stages of the compilation process, the resulting formula is complicated. In order to make it easier to follow, it is shown below in stages, corresponding to the stages at which adjustments are applied. For more information about quality adjustments, see the section on **Quality Assurance**.

$$S_t = \frac{\sum_{d \in S} \left(w_{dy} D_t \right)}{\sum_{d \in S} w_{dy}}$$

where S_t is the value of the IoS (or any section-level output) at time t;
D_t is the value of a division-level (SIC 2-digit) series within the IoS at time t; and
w_{dy} is the aggregate weight (in the previous year) of division D in the index.

$$D_t = \frac{\sum_{g \in D} \left(w_{gy} G_t \right)}{\sum_{g \in D} w_{gy}} + a_{dt} + c_{dt}$$

where D_t is the value of a division-level (SIC 2-digit) series within the IoS at time t;
G_t is the value of a group-level (SIC 4-digit) series within the IoS at time t;
w_{gy} is the aggregate weight (in the previous year) of group G in the index;
a_{dt} is the quality adjustment applied to division D at time t; and
c_{dt} is the quarterly coherence adjustment applied to division D at time t.

$$G_t = \frac{\sum_{i \in G} \left(100 w_i \left(\frac{\left(Q_{it} P_{it} \right) + a_{it}}{\left(Q_{i0} P_{i0} \right) + a_{i0}} \right) \left(\frac{P_{i0}}{P_{it}} \right) \right)}{\sum_{i \in G} w_i} + c_{gt}$$

where G_t is the value of a group-level (SIC 4-digit) series within the IoS at time t;
w_i is the weight in the base year of component i of the index;
$Q_{it}P_{it}$ is the value of a current price input series i at time t;
a_{it} is the quality adjustment applied to the current price input i at time t;
$Q_{i0}P_{i0}$ is the average value of the current price series i in the base year;
a_{i0} is the average value of the adjustment applied to series i in the base year;
P_{i0}/P_{it} is the inverse of the deflator (divide by a ratio = multiply by its inverse); and
c_{gt} is the annual coherence adjustment applied to group G at time t.